Motion of a Projectile 2

Q1.

A particle P is projected with speed $u\,\text{m s}^{-1}$ at an angle of θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t s are denoted by x m and y m respectively.

(a) Starting from the equation of the trajectory given in the List of formulae (MF19), show that

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta).$$ [1]

When $\theta = \tan^{-1}2$, P passes through the point with coordinates (10, 16).

(b) Show that there is no value of θ for which P can pass through the point with coordinates (18, 30). [6]

Q2.

A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is $100\,\text{m s}^{-1}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. The two times at which P's height above the plane is H m differ by 10 s.

(a) Find the value of H. [5]

(b) Find the magnitude and direction of the velocity of P one second before it strikes the plane. [4]

Q3.

A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Use the equation of the trajectory given in the List of formulae (MF19), together with the condition $y = 0$, to establish an expression for the range R in terms of u, θ and g. [2]

(b) Deduce an expression for the maximum height H, in terms of u, θ and g. [2]

It is given that $R = \frac{4H}{\sqrt{3}}$.

(c) Show that $\theta = 60°$. [1]

It is given also that $u = \sqrt{40}\,\text{m s}^{-1}$.

(d) Find, by differentiating the equation of the trajectory or otherwise, the set of values of x for which the direction of motion makes an angle of less than 45° with the horizontal. [4]

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Q4.

A particle P is projected from a point O on a horizontal plane and moves freely under gravity. Its initial speed is $u\,\text{m s}^{-1}$ and its angle of projection is $\sin^{-1}(\frac{4}{5})$ above the horizontal. At time 8 s after projection, P is at the point A. At time 32 s after projection, P is at the point B. The direction of motion of P at B is perpendicular to its direction of motion at A.

Find the value of u. [7]

Q5.

A particle is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane. The particle moves freely under gravity.

(a) Write down the horizontal and vertical components of the velocity of the particle at time T after projection. [2]

At time T after projection, the direction of motion of the particle is perpendicular to the direction of projection.

(b) Express T in terms of u, g and α. [2]

(c) Deduce that $T > \frac{u}{g}$. [1]

Q6.

One end of a light inextensible string of length a is attached to a fixed point O. The other end of the string is attached to a particle P of mass m. The particle P is held vertically below O with the string taut and then projected horizontally. When the string makes an angle of 60° with the upward vertical, P becomes detached from the string. In its subsequent motion, P passes through the point A which is a distance a vertically above O.

(a) The speed of P when it becomes detached from the string is V. Use the equation of the trajectory of a projectile to find V in terms of a and g. [4]

(b) Find, in terms of m and g, the tension in the string immediately after P is initially projected horizontally. [4]

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Q7.

Particles P and Q are projected in the same vertical plane from a point O at the top of a cliff. The height of the cliff exceeds 50 m. Both particles move freely under gravity. Particle P is projected with speed $\frac{35}{2}\,\mathrm{m\,s^{-1}}$ at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. Particle Q is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle β above the horizontal, where $\tan\beta = \frac{1}{2}$. Particle Q is projected one second after the projection of particle P. The particles collide T s after the projection of particle Q.

(a) Write down expressions, in terms of T, for the horizontal displacements of P and Q from O when they collide and hence show that $4uT = 21\sqrt{5}(T+1)$. [4]

(b) Find the value of T. [4]

(c) Find the horizontal and vertical displacements of the particles from O when they collide. [3]

Q8.

A particle P is projected with speed $25\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. After 2 s the speed of P is $15\,\mathrm{m\,s^{-1}}$.

(a) Find the value of $\sin\theta$. [5]

(b) Find the range of the flight. [3]
