Hypothesis Testing 1

Q1.

Metal bolts are produced in large numbers and have lengths which are normally distributed with mean 2.62 cm and standard deviation 0.30 cm.

(i) Find the probability that a random sample of 45 bolts will have a mean length of more than 2.55 cm. [3]

(ii) The machine making these bolts is given an annual service. This may change the mean length of bolts produced but does not change the standard deviation. To test whether the mean has changed, a random sample of 30 bolts is taken and their lengths noted. The sample mean length is m cm. Find the set of values of m which result in rejection at the 10% significance level of the hypothesis that no change in the mean length has occurred. [4]

Q2.

A hospital patient's white blood cell count has a Poisson distribution. Before undergoing treatment the patient had a mean white blood cell count of 5.2. After the treatment a random measurement of the patient's white blood cell count is made, and is used to test at the 10% significance level whether the mean white blood cell count has decreased.

(i) State what is meant by a Type I error in the context of the question, and find the probability that the test results in a Type I error. [4]

(ii) Given that the measured value of the white blood cell count after the treatment is 2, carry out the test. [3]

(iii) Find the probability of a Type II error if the mean white blood cell count after the treatment is actually 4.1. [3]

Q3.

The marks of candidates in Mathematics and English in 2009 were represented by the independent random variables X and Y with distributions $N(28, 5.6^2)$ and $N(52, 12.4^2)$ respectively. Each candidate's marks were combined to give a final mark F, where $F = X + \frac{1}{2}Y$.

(i) Find $E(F)$ and $Var(F)$. [3]

(ii) The final marks of a random sample of 10 candidates from Grinford in 2009 had a mean of 49. Test at the 5% significance level whether this result suggests that the mean final mark of all candidates from Grinford in 2009 was lower than elsewhere. [5]

Hypothesis Testing 1

Q4.

Previous records have shown that the number of cars entering Bampor on any day has mean 352 and variance 121.

(i) Find the probability that the mean number of cars entering Bampor during a random sample of 200 days is more than 354. [4]

(ii) State, with a reason, whether it was necessary to assume that the number of cars entering Bampor on any day has a normal distribution in order to find the probability in part **(i)**. [2]

(iii) It is thought that the population mean may recently have changed. The number of cars entering Bampor during the day was recorded for each of a random sample of 50 days and the sample mean was found to be 356. Assuming that the variance is unchanged, test at the 5% significance level whether the population mean is still 352. [4]

Q5.

An engineering test consists of 100 multiple-choice questions. Each question has 5 suggested answers, only one of which is correct. Ashok knows nothing about engineering, but he claims that his general knowledge enables him to get more questions correct than just by guessing. Ashok actually gets 27 answers correct. Use a suitable approximating distribution to test at the 5% significance level whether his claim is justified. [5]

Q6.

The number of new enquiries per day at an office has a Poisson distribution. In the past the mean has been 3. Following a change of staff, the manager wishes to test, at the 5% significance level, whether the mean has increased.

(i) State the null and alternative hypotheses for this test. [1]

The manager notes the number, N, of new enquiries during a certain 6-day period. She finds that $N = 25$ and then, assuming that the null hypothesis is true, she calculates that $P(N \geqslant 25) = 0.0683$.

(ii) What conclusion should she draw? [2]

Q7.

(i) Deng wishes to test whether a certain coin is biased so that it is more likely to show Heads than Tails. He throws it 12 times. If it shows Heads more than 9 times, he will conclude that the coin is biased. Calculate the significance level of the test. [3]

(ii) Deng throws another coin 100 times in order to test, at the 5% significance level, whether it is biased towards Heads. Find the rejection region for this test. [5]