Permutations & Combinations 2 MS

Q1.

7 (a)	$\frac{10!}{5!4!} = 1260$	M1 A1	[2]	10! or ${}_{10}P_{10}$ seen in num or alone or dividing by 5! 4! only Correct final answer
(b) (i)	${}_8P_4$ or ${}_8C_4 \times 4!$ $= 1680$	M1 A1	[2]	${}_8P_4$ or ${}_8C_4$ oe seen allow extra multiplication Correct answer
(ii)	6C2 × 4! $= 360$ OR 6P4 or $4 \times 3 \times 6 \times 5 = 360$	M1 M1 A1	[3]	6C2 or 6P2 seen multiplied Mult by 4! Correct answer Award full marks
(c)	A B C 1 1 7 = 9C1 × 8C1 × 7C7 (oe) × ${}_3C_1 = 216$ 1 3 5 = 9C1 × 8C3 × 5C5(oe) × 3! = 3024 3 3 3 = 9C3 × 6C3 × 3C3 (oe) = 1680 Total = 4920 ways	M1 M1 M1 A1 A1	[5]	Summing at least two options of 1, 1, 7 or 1, 3, 5 or 3, 3, 3 Mult an option by 3C1 or 3! or 3C3 Any one of the 2nd term being xCy seen mult, fitting with the first (x could be 2, 4, 5, 6 or 8) and correspnding y Any of unsimplified 72, 504 or 1680 seen Correct answer

Q2.

5 (a)	Boys in: 10C1 × 9C3 = 840 ways Boys out: 10C3 × 9C3 = 10080 ways Total = 10920 ways (10900)	M1 B1 A1	[3]	summing two 2-factor products, C or P Any correct option unsimplified Correct final answer
(b) (i)	${}_{12}P_8 = 19,958,400$	B1	[1]	or 20,000,000
(ii)	together: ${}_{11}P_7 = 1663200 \times 2 = 3326400$ Not tog: 19958400 – 3326400 =16,632,000 (16,600,000) OR M at end then not F in 10 × 10P6 × 2=3024000 ways not at end in 10 × 9 × 10P6 = 13608000 ways Total = 16,632,000 ways	B1 M1 A1 M1 B1 A1	[3]	${}_{11}P_7$ seen 19958400 or their (i) – their together (must be >0) correct final answer summing options for M at end and M not at end one correct option correct final answer
(iii)	8! × 5 = 201600 ways	B1 M1 A1	[3]	8! seen mult by equivalent of integer ≥ 1 Mult by 5 Correct answer SR 8! × 5!=4838400 B2

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Q3.

6	(a)	twins in: ${}_6C_2$ twins out: ${}_5C_2 \times {}_6C_2$ Total = 15 + 150 = 165 OR all: ${}_7C_2 \times {}_6C_2$ one twin: $2 \times {}_5C_1 \times {}_6C_2$ Total = 315 − 150 = 165	B1 M1 A1 B1 M1 A1	**3**	${}_6C_2$ alone or ${}_5C_2$ multiplied seen or implied Summing two cases Correct final answer ${}_7C_2 \times {}_6C_2$ alone or ${}_5C_1$ multiplied seen or implied $2 \times {}_5C_1 \times {}_6C_2$ seen, subtracted Correct final answer
	(b)				
	(i)	ends in 2, 6 or 8: 6!/2! (= 360) ways ends in 4: 6! (= 720) ways Total = 3 × 360 + 720 = 1800 ways	B1 B1 M1 A1	**4**	Correct option for ending with 2 or 6 or 8.6!/2! seen anywhere, not multiplied Correct option for ending in 4 Summing 3 or 4 even options Correct final answer
		OR_1 all: 7!/2! (= 2520) ways ends in 1 or 7: 6!/2! (= 360) ways Total = 2520 − 2 × 360 = 1800	B1 B1 M1 A1		7!/2! seen anywhere, not multiplied 6!/2! seen, subtracted Subtract 2 odd options from total options Correct final answer
		OR_2 (4_A, 4_B) final digit: 5 ways other digits: 6! ways and ÷ by 2! Total = 5 × 360 = 1800	B1 B1 M1 A1		5 seen, multiplied 6! seen and divide by 2! at some stage Multiplying their two numbers Correct final answer
	(ii)	$5 \times 4 \times 3 \times 2$ or ${}_5P_4$ or ${}_5C_4 \times 4!$ or 5! or ${}_5P_5$ or ${}_6P_5 \div 6$ = 120 ways	M1 A1	**2**	One of these oe Correct final answer
	(c)	$\left(\frac{2}{3}\right)^7$ $= \frac{128}{2187}$ (0.0585)	M1 M1 A1	**3**	2/3 seen multiplied 7 probabilities multiplied together Correct final answer

Q4.

6	(i)	$4! \times 3! \times 5! \times 2! \times 4! = 829440$	B1 B1 B1	**[3]**	4!, 3!, 5!, 2 seen multiplied 1, not in denominator Mult by 4! Correct answer
	(ii)	$8! \times 9 \times 8 \times 7 \times 6 \times 5 \times 4$ $= 2438553600$ (2.44×10^9)	B1 B1 B1	**[3]**	8! seen multiplied 1 Mult by ${}_9P_6$ Correct answer
	(iii)	8C3 × 5C3 × 2C2 = 560	B1 B1 B1	**[3]**	8C3 seen mult 5C3 seen mult Correct answer

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Q5.

6	(i)	H J O 1. 28 2 = 4C2×9C8×2C2 = 54 3 7 2 = 4C3×9C7×2C2 = 144 4 6 2 = 4C4×9C6×2C2 = 84 Total = 282 ways	M1 M1 A1 A1	[4]	Mult 3 combs, 2C2 may be implied 4Cx×9Cy×2Cz Summing 2 or 3 three-factor options 2 options correct unsimplified Correct answer
	(ii)	4! × 6! × 2! × 3! = 207360 (207000)	M1 M1 A1	[3]	4! × 6! × 2! oe seen multiplied by int ≥ 1 3! seen mult by int ≥ 1 Correct answer
	(iii)	8 J and O trees in 8! = 40320 ways 9 gaps × 8 × 7 × 6 = 121,927,680 (122,000,000)	B1 M1 A1	[3]	8! seen mult by int ≥ 1 no division 9P4 oe or 7P4 or 8P4 seen mult by int ≥ 1 no division Correct answer
	(i)	SR 4C2×9C2×2C2×9C6	M1		
	(ii)	SR $\frac{4!\times6!\times2!}{4!\times6!\times2!}$ or 3! or both M1	M1		

Q6.

6	(i)	$\frac{6!}{2!}=360$	B1 B1	2	6! Seen alone Dividing by 2! only
	(ii)	$\frac{4!}{2!}\times\frac{4!}{3!}$ = 48	B1 B1 B1	3	4! seen mult Dividing by 2! or 3! (Mult by 4 implied B1B1) Correct answer
	(iii)	1N and 1A: N A xx in $^3C_2$ = 3 ways	M1 A1	2	^3C_x or $^xC_2$ seen alone Correct answer
	(iv)	0 A : Nxxx = 1 way 2 As: NAAx in $^3C_1$ = 3 ways 3 As: NAAA in 1 way Total = 8 ways	M1 M1 A1	3	Finding ways with 0 or 2 or 3 As Summing 3 or 4 options Correct answer

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Q7.

5 **(i)**	$5! \times 3!$ or $6!$	B1		$5!$ or $3!$ or $6!$ oe seen mult or alone
	$= 720$	B1	**2**	Correct final answer
(ii)	3**4, 3**8, 4**8	M1		considering at least 2 types of 4-figure options ending with 4 or 8 and starting with 3 or 4
		B1		One option correct unsimplified can be implied
	$= 5 \times 4 + 5 \times 4 + 5 \times 4 = 60$	A1	**3**	Correct final answer
(iii)	5, *5, **5,	M1		Appreciating that the number must end in 5 (can be implied)
	$= 1 + 7 + 7^2$	M1		summing numbers ending in 5 with at least 2 different numbers of digits
	$= 57$	A1	**3**	Correct final answer

Q8.

7 **(i)** **(a)**	6!	M1		Seen in a single term expression as numerator
	(×) 4! **OR** (×) 4 × 3	M1		Seen in a single term expression as numerator (denominator may be 1)
	÷ 2!2!3! **OR** ÷ 2!3!	M1		Seen in a single term expression as denominator
	Total 720 ways	A1	**4**	Correct ans
(i) **(b)**	$1*******3 = \frac{7!}{3!2!} = 420$	B1		$\frac{7!}{3!2!}$ seen oe
	3*******1= 420 3*******3= 420	M1		Attempting to evaluate and sum at least 2 of 1***3, 3***1, 3***3
	Total = 1260 ways	A1	**3**	Correct ans
(ii) **(a)**	$5 \times 4 \times 3 = 60$ ways ($^5P_3$)	M1		$^5P_3$ or $^5C_3 \times 3!$ (can be implied)
		A1	**2**	Correct ans
(ii) **(b)**	2** in 212, 213, 214, 216, 221, 223, 224, 226, 231, 232, 233, 234, 236, 241, 242, 243, 246 261, 262, 263, 264, 266	M1		Listing attempt starting with 2, at least 10 correct entries
	Total = 22 ways	A1	**2**	Correct ans
	Alternative Methods: $3 \times {}^4C_1 + 2 \times {}^5C_1$	M1		$p \times {}^4C_1 + q \times {}^5C_1$, oe $p + q > 2$
	OR $^5P_2 + {}^2C_1$	**OR** M1		$^5P_2$ seen
	OR $^4P_2 + 2 \times {}^4P_1 + {}^2C_1$	**OR** M1		Any 2 terms added

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Q9.

7	(i)	W(8) M(5) 4 2 = ${}^8C_4 \times {}^5C_2 = 700$ 5 1 = ${}^8C_5 \times {}^5C_1 = 280$ 6 0 = ${}^8C_6 \times {}^5C_0 = 28$ Total = 1008	M1 M1 A1 A1	**4**	Mult 2 combs, ${}^8C_x \times {}^5C_y$ Summing 2 or 3 options 2 correct options unsimplified Correct answer
	(ii)	M1 and MMWWW = ${}^3C_2 \times {}^8C_3 = 168$ M2 and MMWWW = ${}^3C_2 \times {}^8C_3 = 168$ Neither and MMMWWW = ${}^3C_1 \times {}^8C_3 = 56$ Total = 392	M1 B1 A1	**3**	Summing 3 options One correct option Correct answer
		OR total, no restrictions = ${}^5C_3 \times {}^8C_3 = 560$ M1M2 and MWWW = ${}^3C_1 \times {}^8C_3 = 168$ 560 – 168 = 392	M1 B1 A1		Subt 2 men together from no restrictions One correct of 560 or 168 Correct answer