Functions 1

Q1.

The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $\mathrm{f}(x)$ in the form $a(x-b)^2 - c$. [3]

(ii) State the range of f. [1]

(iii) Find the set of values of x for which $\mathrm{f}(x) < 21$. [3]

The function g is defined by $\mathrm{g} : x \mapsto 2x + k$ for $x \in \mathbb{R}$.

(iv) Find the value of the constant k for which the equation $\mathrm{gf}(x) = 0$ has two equal roots. [4]

Q2.

The functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto 4x - 2x^2,$$
$$\mathrm{g} : x \mapsto 5x + 3.$$

(i) Find the range of f. [2]

(ii) Find the value of the constant k for which the equation $\mathrm{gf}(x) = k$ has equal roots. [3]

Q3.

The function $\mathrm{f} : x \mapsto 2x^2 - 8x + 14$ is defined for $x \in \mathbb{R}$.

(i) Find the values of the constant k for which the line $y + kx = 12$ is a tangent to the curve $y = \mathrm{f}(x)$. [4]

(ii) Express $\mathrm{f}(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(iii) Find the range of f. [1]

The function $\mathrm{g} : x \mapsto 2x^2 - 8x + 14$ is defined for $x \geqslant A$.

(iv) Find the smallest value of A for which g has an inverse. [1]

(v) For this value of A, find an expression for $\mathrm{g}^{-1}(x)$ in terms of x. [3]

Functions 1

Q4.

Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 2x + 3,$$
$$g : x \mapsto x^2 - 2x.$$

Express $gf(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [5]

Q5.

The function f is defined by

$$f(x) = x^2 - 4x + 7 \quad \text{for} \quad x > 2.$$

(i) Express $f(x)$ in the form $(x-a)^2 + b$ and hence state the range of f. [3]

(ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

The function g is defined by

$$g(x) = x - 2 \quad \text{for} \quad x > 2.$$

The function h is such that $f = hg$ and the domain of h is $x > 0$.

(iii) Obtain an expression for $h(x)$. [1]

Q6.

Functions f and g are defined by

$$f : x \mapsto 3x - 4, \quad x \in \mathbb{R},$$
$$g : x \mapsto 2(x-1)^3 + 8, \quad x > 1.$$

(i) Evaluate $fg(2)$. [2]

(ii) Sketch in a single diagram the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

(iii) Obtain an expression for $g'(x)$ and use your answer to explain why g has an inverse. [3]

(iv) Express each of $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x. [4]

Functions 1

Q7.

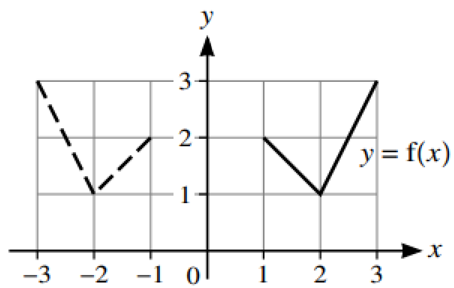
The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 1 + \mathrm{f}(\frac{1}{2}x)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [4]

Q8.

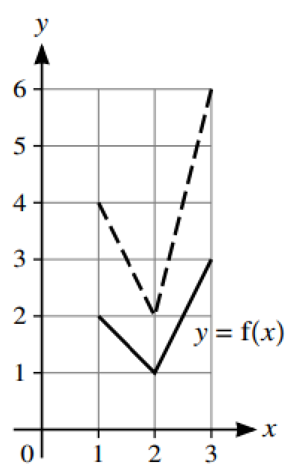
In each of parts **(a)**, **(b)** and **(c)**, the graph shown with solid lines has equation $y = \mathrm{f}(x)$. The graph shown with broken lines is a transformation of $y = \mathrm{f}(x)$.

(a)



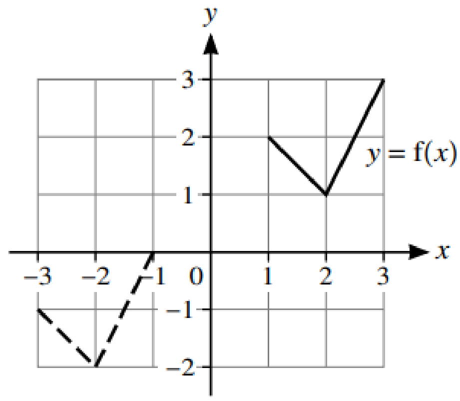
State, in terms of f, the equation of the graph shown with broken lines. [1]

(b)



State, in terms of f, the equation of the graph shown with broken lines. [1]

(c)



State, in terms of f, the equation of the graph shown with broken lines. [2]

Q9.

(a) The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 2\mathrm{f}(x-1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

(b) The curve $y = \sin 2x - 5x$ is reflected in the y-axis and then stretched by scale factor $\frac{1}{3}$ in the x-direction.

Write down the equation of the transformed curve. [2]
