Equilibrium of a Rigid Body 1 MS

Q1.

3	Volume of cylinder $= \pi \times 0.22 \times 0.7\ (= 0.028\pi)$ **AND** Volume of hemisphere $= 2\pi \times \frac{0.2^3}{3}\left(= 0.0053333\pi\right)$	**B1**	Both volumes required for **B1**
	Distance of centre of mass from object base $= 0.7 - 3 \times \frac{0.2}{8}\left(= 0.625\right)$	**B1**	
	$x\left(\pi \times 0.2^2 \times 0.7 - 2\pi \times \frac{0.2^3}{3}\right) + \left(0.7 - 3 \times \frac{0.2}{8}\right) \times 2\pi \times \frac{0.2^3}{3} = 0.35 \times 0.028\pi$	**M1A1**	Take moments about the plane face
	$x = 0.285$ m	**A1**	
		5	

Q2.

6(i)	From $AB = 0.2$	**B1**	
	From $BC = 0.1$	**B1**	
		2	
6(ii)	$\tan\theta = \frac{0.1}{0.2}$	**M1**	θ is the angle between AB and the horizontal
	$\theta = 26.6°$	**A1**	
		2	
6(iii)	$12\cos 26.6 \times 0.3 = W \times 0.2$	**M1A1**	Take moments about B. (W is the weight of the lamina)
	$W = 16.1\ N$	**A1**	
		3	

Q3.

7(i)	$X = \frac{2r}{\pi}$	**B1**	X = distance of centre of mass of the arc from ABC
	$0.8 \times 0.1 = \pi r \times \frac{2r}{\pi}$	**M1**	Take moments about ABC
	$r = 0.2$	**A1**	
		3	
7(ii)	$AC = 0.8 + 2 \times 0.2 - 0.2\pi\ (= 0.57168\ldots)$	**B1**	
	$0.1W = 7AC$	**M1**	AC must be a numerical value. Take moments about A
	$W = 40(0.)\ N$	**A1**	
		3	

7(iii)	$(0.8 - 0.2\pi + 0.2)\ [= 0.37168\ldots]$	**B1**	
	$0.8Y = (0.8 - 0.2\pi) \times \frac{(0.8 - 0.2\pi)}{2} + (0.2\pi) \times (0.8 - 0.2\pi + 0.2)$	**M1A1**	
	$Y = 0.310(338)$	**A1**	
	$\tan\theta = \frac{0.1}{0.310338}$	**M1**	
	$\theta = 17.9$	**A1**	Allow 17.8

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Q4.

1	Conservation of momentum at $\frac{h}{4}$	**B1**	
	$\frac{5\times h}{4}=3\times 0.2$	**M1**	Take moments about A
	$(h=)$ 0.48 m	**A1**	
		3	

Q5.

7(i)	Rectangle: Area $=1.2\times 1.8=2.16,\ y=\frac{1.8}{2}=0.9$	**B1**	
	Triangle(s): Area $=1.2\times\frac{1.8}{2}=1.08,\ y=\frac{1.8}{3}=0.6$	**B1**	
	$(2.16+1.08)Y=2.16\times 0.9+1.08\times 0.6$	**M1**	Take moments about AD
	$Y=0.8$ m	**A1**	AG
		4	

7(ii)	$AG\sin 30=0.8$	**M1**	Use Trigonometry of a right angled triangle
	$AG=1.6$ m	**A1**	
		2	
7(iii)	AD makes an angle of 40° or 20° with the vertical	**B1**	
	$W\times AG\sin 10=7\times 2.4\cos 40$	**M1**	Take moments about A
	$W=46.3$ N	**A1**	
	$W\times AG\sin 10=7\times 2.4\cos 20$	**M1**	Take moments about A
	$W=56.8$ N	**A1**	
		5	

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Q6.

<table>
<tr><td>4(a)</td><td>
<table>
<tr><th></th><th>Area</th><th>Centre of mass from BC</th><th>Centre of mass from DC</th></tr>
<tr><td>Square</td><td>100</td><td>5</td><td>5</td></tr>
<tr><td>Triangle</td><td>½ x. 15/2</td><td>$\frac{1}{3}x$</td><td>$\frac{5}{2}$</td></tr>
<tr><td>Shape ABEFD</td><td>$100-\frac{15}{4}x$</td><td>$\bar{x}$</td><td>$\bar{y}$</td></tr>
</table>
Take moments about BC:
$\left(100-\frac{15}{4}x\right)\sigma\,.\,\bar{x}=500\sigma-\frac{15}{4}x\sigma.\frac{1}{3}x$
(M1 for all terms present)</td><td>M1</td></tr>
<tr><td></td><td>$\bar{x}=\frac{400-x^2}{80-3x}$ AG</td><td>A1</td></tr>
<tr><td></td><td>Take moments about DC:
$\left(100-\frac{15}{4}x\right).\,\bar{y}\,=100\times5-\frac{15}{4}x.\frac{5}{2}$</td><td>M1</td></tr>
<tr><td></td><td>$\bar{y}\,=\frac{800-15x}{160-6x}$</td><td>A1</td></tr>
<tr><td></td><td></td><td>4</td></tr>
<tr><td>4(b)</td><td>Use condition: $\bar{x}\geqslant x$</td><td>B1</td></tr>
<tr><td></td><td>$2x^2-80x+400\geqslant0$</td><td>M1</td></tr>
<tr><td></td><td>$x=20-10\sqrt{2}$</td><td>A1</td></tr>
<tr><td></td><td></td><td>3</td></tr>
</table>