Functions 2 MS

Q1.

10 **(i)** Range is (y) $\geq c^2 + 4c$	B1	Allow >
$x^2 + 4x = (x+2)^2 - 4$	M1	**OR** $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x + 4 = 0$
(Smallest value of c is) -2	A1 [3]	-2 with no (wrong) working gets B2
(ii) $5a + b = 11$	B1	
$(a+b)^2 + 4(a+b) = 21$	B1	
$(11 - 5a + a)^2 + 4(11 - 5a + a) = 21$	M1	**OR** corresponding equation in b
$(8)(2a^2 - 13a + 18) = (8)(2a-9)(a-2) = 0$	M1	**OR** $(8)(2b+23)(b-1) = 0$
$a = \frac{9}{2}, 2$ OR $b = \left(-\frac{23}{2}\right), 1$	A1 A1 [6]	A1 for either a or b correct. Condone 2nd value. Spotted solution scores only B marks.
Alt. **(ii)** Last 5 marks		**Alt. (ii)** Last 4 marks
$f^{-1}(x) = \sqrt{x+4} - 2$ B1		$(a+b+7)(a+b-3) = 0$ M1A1
$g(1) = f^{-1} = (21)$ used M1		(Ignore solution involving $a + b = -7$)
$a + b = \sqrt{25} - 2 = 3$ A1		
Solve $a + b = 3$, $5a + b = 11$ M1		Solve $a + b = 3$, $5a + b = 11$ M1
$a = 2$, $b = 1$ A1		$a = 2$, $b = 1$ A1

Q2.

10 **(i)** $-5 \leqslant f(x) \leqslant 4$ For f(x) allow x or y; allow <, [–5, 4], (–5,4)	**B1** **[1]**	Allow less explicit answers (eg $-5 \rightarrow 4$)
(ii) $f^{-1}(x)$ approximately correct (independent of f)	**B1**	
Closed region between (1, 1) and (4, 4); line reaches x-axis	**DB1** **[2]**	Ignore line $y = x$
(iii) LINE: $f^{-1}(x) = \frac{1}{3}(x+2)$	**B1**	Allow $y =$ ..... but must be a function of x
for $-5 \leqslant x \leqslant 1$	**B1B1**	cao but allow <
CURVE: $5 - y = \frac{4}{x}$ OR $x = 5 - \frac{4}{y}$	**M1**	
$f^{-1}(x) = 5 - \frac{4}{x}$ oe	**A1**	cao
for $1 < x \leqslant 4$	**B1** **[6]**	cao but allow < or <

Functions 2 MS

Q3.

10 f : $x \mapsto 2x - 3,\ x \in \mathbb{R}$, g : $x \mapsto x^2 + 4x,\ x \in \mathbb{R}$.		
(i) $ff = 2(2x - 3) - 3$ Solves $= 11 \rightarrow x = 5$ (or $2x-3 = 11, x = 7.\ 2x-3=7 \rightarrow x = 5$)	M1 A1 [2]	Either forms ff correctly, or solves 2 equations co
(ii) min at $x = -2$ $\rightarrow$ Range $\geqslant -4$	M1 A1 [2]	Any valid method – could be guesswork.
(iii) $x^2 + 4x - 12\ \ (> 0)$ $\rightarrow x = 2$ or -6 $\rightarrow x < -6\ ,\ x > 2.$	M1 A1 A1 [3]	Makes quadratic = 0 + 2 solutions Correct limits – even if $>,<,\geqslant,\leqslant,=$ co
(iv) $gf(x) = (2x - 3)^2 + 4(2x - 3) = p$ $\rightarrow 4x^2 - 4x - 3 - p = 0$ Uses "$b^2 - 4ac$" $16 = 16(-3 - p)$ $\rightarrow p = -4$	B1 M1 A1 [3]	co unsimplified Use of discriminant co
(v) -2	B1 [1]	co
(vi) $y = (x+2)^2 - 4$ $\sqrt{y+4} = x + 2$ $h^{-1}(x) = \sqrt{x+4} - 2$	B2,1 M1 A1 [4]	-1 for each error Correct order of operations co with x, not y. $\pm$ left A0.

Q4.

6	**(i)**	Attempt to find $(f^{-1})^{-1}$	**M1**	
		$2xy = 1 - 5x$ or $\frac{1}{2x} = y + \frac{5}{2}$ Allow 1 sign error	**A1**	Or with x/y transposed.
		$x = \frac{1}{2y+5}$ oe Allow 1 sign error (total)	**A1**	Or with x/y transposed. Allow $x = \frac{\frac{1}{2}}{y + \frac{5}{2}}$.
		$(f(x)) = \frac{1}{2x+5}$ for $x \geqslant -\frac{9}{4}$ **(Allow** $-\frac{9}{4} \leqslant x \leqslant \infty$)	**A1 B1** **[5]**	Allow $\frac{\frac{1}{2}}{x + \frac{5}{2}}$. Condone $x > \frac{-9}{4}$, $(\frac{-9}{4}, \infty)$ (etc.)
	(ii)	$f^{-1}\left(\frac{1}{x}\right) = \frac{1 - \frac{5}{x}}{\frac{2}{x}}$ $\frac{x-5}{2}$ or $\frac{1}{2}x - \frac{5}{2}$	**M1** **A1** **[2]**	Reasonable attempt to find $f^{-1}\left(\frac{1}{x}\right)$.

Functions 2 MS

Q5.

1	$f : x \mapsto 10 - 3x,\ g : x \mapsto \dfrac{10}{3-2x}$,		
	$ff(x) = 10 - 3(10 - 3x)$	**B1**	Correct unsimplified expression
	$gf(2) = \dfrac{10}{3-2(10-3(2))}\ (= -2)$	**B1**	Correct unsimplified expression with 2 in for x
	$x = 2$	**B1** [3]	

Q6.

9	$f : x \mapsto \dfrac{2}{3-2x}$ $g : x \mapsto 4x + a$,		
9(i)	$y = \dfrac{2}{3-2x} \rightarrow y(3-2x) = 2 \rightarrow 3 - 2x = \dfrac{2}{y}$	**M1**	Correct first 2 steps
	$\rightarrow 2x = 3 - \dfrac{2}{y} \rightarrow f^{-1}(x) = \dfrac{3}{2} - \dfrac{1}{x}$	**M1 A1**	Correct order of operations, any correct form with $f(x)$ *or* $y =$
	Total:	**3**	
9(ii)	$gf(-1) = 3\ f(-1) = \dfrac{2}{5}$	**M1**	Correct first step
	$\dfrac{8}{5} + a = 3 \rightarrow a = \dfrac{7}{5}$	**M1 A1**	Forms an equation in a and finds a, OE
			(or $\dfrac{8}{3-2x} + a = 3$, **M1** Sub and solves **M1**, **A1**)
	Total:	**3**	
9(iii)	$g^{-1}(x) = \dfrac{x-a}{4} = f^{-1}(x)$	**M1**	Finding $g^{-1}(x)$ and equating to their $f^{-1}(x)$ even if $a = 7/5$
	$\rightarrow x^2 - x(a+6) + 4(=0)$	**M1**	Use of $b^2 - 4ac$ on a quadratic with a in a coefficient
	Solving $(a+6)^2 = 16$ *or* $a^2 + 12a + 20(=0)$	**M1**	Solution of a 3 term quadratic
	$\rightarrow a = -2$ or -10	**A1**	
	Total:	**4**	

Q7.

6(a)	$f(x) = (x-1)^2 + 4$	**B1**	
	$g(x) = (x+2)^2 + 9$	**B1**	
	$g(x) = f(x+3) + 5$	**B1 B1**	B1 for each correct element. Accept $p = 3, q = 5$
		4	

Functions 2 MS

6(b)	Translation or Shift	B1	
	$\begin{pmatrix}-3\\5\end{pmatrix}$ or acceptable explanation	B1 FT	If given as 2 single translations both must be described correctly e.g. $\begin{pmatrix}-3\\0\end{pmatrix}$ & $\begin{pmatrix}0\\5\end{pmatrix}$ FT from *their* $\mathrm{f}(x+p)+q$ *or their* $\mathrm{f}(x)\to\mathrm{g}(x)$ Do not accept $\begin{pmatrix}1\\4\end{pmatrix}$ or $\begin{pmatrix}-2\\9\end{pmatrix}$
		2	

Q8.

5(a)	$2[\{(x-2)^2\}\ \{+3\}]$	**B1 B1**	B1 for $a=2$, B1 for $b=3$. $2(x-2)^2+6$ gains B1B0
		2	
5(b)	{Translation} $\begin{pmatrix}\{2\}\\\{3\}\end{pmatrix}$ OR {Stretch} {y direction} {factor 2}	**B2,1,0**	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
	{Stretch} {y direction} {factor 2} OR {Translation} $\begin{pmatrix}\{2\}\\\{6\}\end{pmatrix}$	**B2,1,0**	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
		4	

Q9

5(a)	Three points at the bottom of their transformed graph plotted at $y=2$	**B1**	All 5 points of the graph must be connected.
	Bottom three points of /\/\ at $x=0$, $x=1$ & $x=2$	**B1**	Must be this shape.
	All correct	**B1**	Condone extra cycles outside $0\leqslant x\leqslant 2$.
		3	**SC:** If B0 B0 scored, B1 available for /\ in one of correct positions **or** all 5 points correctly plotted and not connected **or** correctly sized shape in the wrong position.
5(b)	$[\mathrm{g}(x)=]\ \mathrm{f}(2x)+1$	**B1 B1**	Award marks for their final answer as follows: f(2x) B1, + 1 B1. Condone $y=$ or $f(x)=$.
		2	