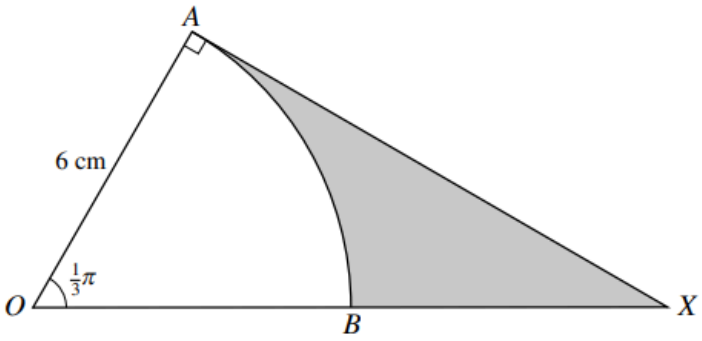
Circular Measure 1

Q1.



In the diagram, AB is an arc of a circle, centre O and radius 6 cm, and angle $AOB = \frac{1}{3}\pi$ radians. The line AX is a tangent to the circle at A, and OBX is a straight line.

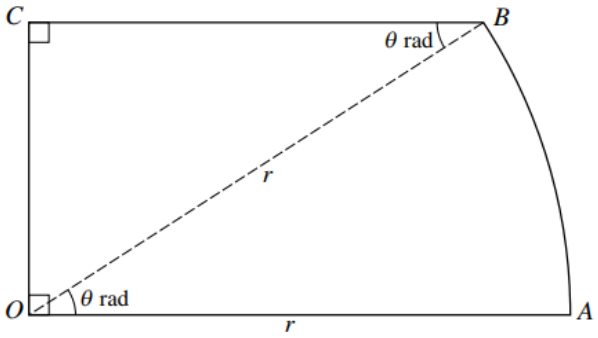
(i) Show that the exact length of AX is $6\sqrt{3}$ cm. [1]

Find, in terms of π and $\sqrt{3}$,

(ii) the area of the shaded region, [3]

(iii) the perimeter of the shaded region. [4]

Q2.



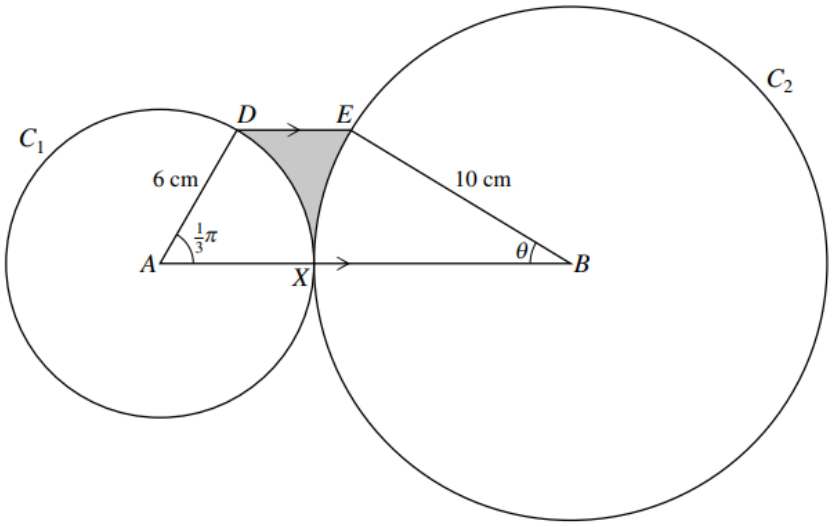
The diagram represents a metal plate $OABC$, consisting of a sector OAB of a circle with centre O and radius r, together with a triangle OCB which is right-angled at C. Angle $AOB = \theta$ radians and OC is perpendicular to OA.

(i) Find an expression in terms of r and θ for the perimeter of the plate. [3]

(ii) For the case where $r = 10$ and $\theta = \frac{1}{5}\pi$, find the area of the plate. [3]

Circular Measure 1

Q3.

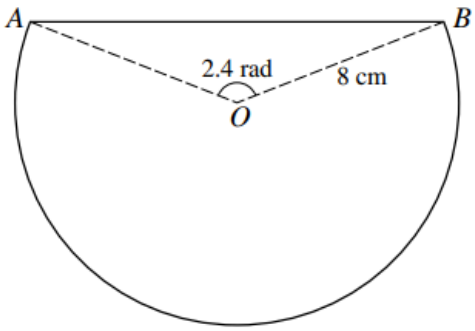


The diagram shows a circle C_1 touching a circle C_2 at a point X. Circle C_1 has centre A and radius 6 cm, and circle C_2 has centre B and radius 10 cm. Points D and E lie on C_1 and C_2 respectively and DE is parallel to AB. Angle $DAX = \frac{1}{3}\pi$ radians and angle $EBX = \theta$ radians.

(i) By considering the perpendicular distances of D and E from AB, show that the exact value of θ is $\sin^{-1}\left(\frac{3\sqrt{3}}{10}\right)$. [3]

(ii) Find the perimeter of the shaded region, correct to 4 significant figures. [5]

Q4.



The diagram shows a metal plate made by removing a segment from a circle with centre O and radius 8 cm. The line AB is a chord of the circle and angle $AOB = 2.4$ radians. Find

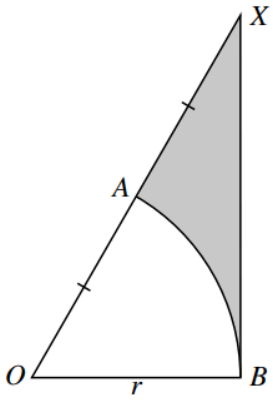
(i) the length of AB, [2]

(ii) the perimeter of the plate, [3]

(iii) the area of the plate. [3]

Circular Measure 1

Q5.



In the diagram, AB is an arc of a circle with centre O and radius r. The line XB is a tangent to the circle at B and A is the mid-point of OX.

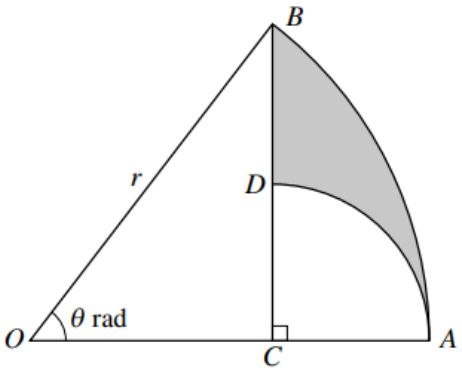
(i) Show that angle $AOB = \frac{1}{3}\pi$ radians. [2]

Express each of the following in terms of r, π and $\sqrt{3}$:

(ii) the perimeter of the shaded region, [3]

(iii) the area of the shaded region. [2]

Q6.



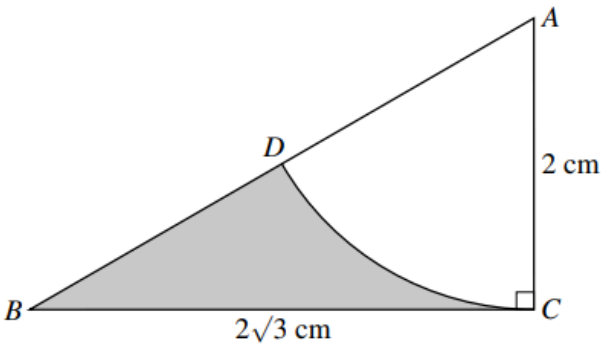
The diagram shows a sector OAB of a circle with centre O and radius r. Angle AOB is θ radians. The point C on OA is such that BC is perpendicular to OA. The point D is on BC and the circular arc AD has centre C.

(i) Find AC in terms of r and θ. [1]

(ii) Find the perimeter of the shaded region ABD when $\theta = \frac{1}{3}\pi$ and $r = 4$, giving your answer as an exact value. [6]

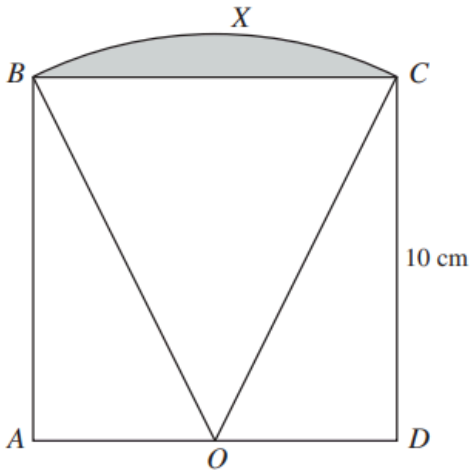
Circular Measure 1

Q7.



In the diagram, D lies on the side AB of triangle ABC and CD is an arc of a circle with centre A and radius 2 cm. The line BC is of length $2\sqrt{3}$ cm and is perpendicular to AC. Find the area of the shaded region BDC, giving your answer in terms of π and $\sqrt{3}$. [4]

Q8.



The diagram shows a square $ABCD$ of side 10 cm. The mid-point of AD is O and BXC is an arc of a circle with centre O.

(i) Show that angle BOC is 0.9273 radians, correct to 4 decimal places. [2]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [2]
