Trigonometry 2

Q1.

(i) Prove the identity $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} \equiv \dfrac{\tan\theta - 1}{\tan\theta + 1}$. [1]

(ii) Hence solve the equation $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \dfrac{\tan\theta}{6}$, for $0° \leqslant \theta \leqslant 180°$. [4]

Q2.

(i) Express the equation $3\sin\theta = \cos\theta$ in the form $\tan\theta = k$ and solve the equation for $0° < \theta < 180°$. [2]

(ii) Solve the equation $3\sin^2 2x = \cos^2 2x$ for $0° < x < 180°$. [4]

Q3.

(i) Show that the equation $\dfrac{4\cos\theta}{\tan\theta} + 15 = 0$ can be expressed as

$$4\sin^2\theta - 15\sin\theta - 4 = 0.$$ [3]

(ii) Hence solve the equation $\dfrac{4\cos\theta}{\tan\theta} + 15 = 0$ for $0° \leqslant \theta \leqslant 360°$. [3]

Q4.

(i) Prove the identity $\left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right)^2 \equiv \dfrac{1 - \cos x}{1 + \cos x}$. [4]

(ii) Hence solve the equation $\left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right)^2 = \frac{2}{5}$ for $0 \leqslant x \leqslant 2\pi$. [3]

Q5.

(a) Solve the equation $\sin^{-1}(3x) = -\frac{1}{3}\pi$, giving the solution in an exact form. [2]

(b) Solve, by factorising, the equation $2\cos\theta\sin\theta - 2\cos\theta - \sin\theta + 1 = 0$ for $0 \leqslant \theta \leqslant \pi$. [4]

Trigonometry 2

Q6.

The function f is defined by $f : x \mapsto 4\sin x - 1$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) State the range of f. [2]

(ii) Find the coordinates of the points at which the curve $y = f(x)$ intersects the coordinate axes. [3]

(iii) Sketch the graph of $y = f(x)$. [2]

(iv) Obtain an expression for $f^{-1}(x)$, stating both the domain and range of f^{-1}. [4]

Q7.

(i) Prove the identity $\dfrac{1+\cos\theta}{1-\cos\theta} - \dfrac{1-\cos\theta}{1+\cos\theta} \equiv \dfrac{4}{\sin\theta\tan\theta}$. [4]

(ii) Hence solve, for $0° < \theta < 360°$, the equation

$$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3.$$ [3]

Q8.

(i) Show that $3\sin x\tan x - \cos x + 1 = 0$ can be written as a quadratic equation in $\cos x$ and hence solve the equation $3\sin x\tan x - \cos x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [5]

(ii) Find the solutions to the equation $3\sin 2x\tan 2x - \cos 2x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [3]

Q9.

(i) Show that $\cos^4 x \equiv 1 - 2\sin^2 x + \sin^4 x$. [1]

(ii) Hence, or otherwise, solve the equation $8\sin^4 x + \cos^4 x = 2\cos^2 x$ for $0° \leqslant x \leqslant 360°$. [5]

Q10.

(i) Express the equation $\sin 2x + 3\cos 2x = 3(\sin 2x - \cos 2x)$ in the form $\tan 2x = k$, where k is a constant. [2]

(ii) Hence solve the equation for $-90° \leqslant x \leqslant 90°$. [3]
