Integration 1 MS

Q1.

6 $\frac{dy}{dx} = 3\sqrt{x} - 6 \quad (9, 2)$		
(i) $y = \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - 6x\,(+c)$	B2,1	Loses 1 for each error – ignore $+c$
$(9, 2) \quad 2 = 54 - 54 + c$	M1	Uses (9, 2) with integration to find c.
$\rightarrow c = 2.$	A1	co.
	[4]	
(ii) $\frac{dy}{dx} = 0 \rightarrow x = 4$	B1	Ignore any y-value
$\frac{d^2y}{d^2x} = \frac{3x^{-\frac{1}{2}}}{2}$	M1 A1	Any valid method. co.
$\rightarrow$ +ve (or ¾) Minimum	[3]	

Q2.

2 $y = \frac{a}{x}$		
Volume $= \pi \int \left(\frac{a^2}{x^2}\right) dx \quad = (\pi)\left[\frac{-a^2}{x}\right]$	M1 B1	For using correct formula with π. For correct integration of x^{-2} only
Use of limits 1 to 3 $\rightarrow \frac{2\pi a^2}{3}$	M1	Must be using y^2 or πy^2.
Equates to $24\pi \rightarrow a = 6$	A1	Co, allow ±6.
	[4]	

Q3.

1	$\int \left(x + \frac{1}{x}\right)^2 dx$			
	$= \frac{x^3}{3} - \frac{1}{x} + 2x + (c)$	B1 × 3	[3]	co. Omission of middle term of expansion can still get 2/3.

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Q4.

11	$y = \dfrac{9}{2-x}$			
	(i) $\dfrac{dy}{dx} = -9(2-x)^{-2} \times -1$	B1 B1		Without the "$\times -1$" Indep. With the "$\times -1$". Indep.
	$\dfrac{9}{(2-x)^2} \neq 0$. No turning points.	B1√	[3]	√ provided of form $k \div (2-x)^2$.
	(ii) $V = \pi \int \dfrac{81}{(2-x)^2} dx$			
	$\int y^2 dx = -81(2-x)^{-1} \div (-1)$	B1 B1		Answer without the "$\div -1$ including π For "$\div -1$".
	Use of limits 0 to 1	M1		Uses both limits in an integral of y^2 – if "0" ignored, M0.
	$\rightarrow \dfrac{81\pi}{2}$ (or 127)	A1	[4]	co (If π omitted – max 3/4)
	(iii) $\dfrac{9}{2-x} = x + k$	M1		Elimination of y
	$\rightarrow x^2 - 2x + kx - 2k + 9 = 0$ Uses $b - 4ac$ $\rightarrow k^2 + 4k - 32$	M1		Uses discriminant
	$\rightarrow$ end-points of 4 and -8	A1		End-values correct.
	Range for 2 points of intersection $\rightarrow k < -8$, $k > 4$.	A1	[4]	Accept $\leq$, $\geq$.

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Q5.

11	**(i)**	$A = (0, 1)$ $B = (5, \tfrac{1}{2})$ $y - 1 = -\dfrac{1}{10}(x - 0)$ $y = -\dfrac{1}{10}x + 1$	B1 B1 M1 A1	[4]	 ft *their A,B* **AG**
	(ii)	Curve: $(\pi)\int_0^5 (3x+1)^{-1/2}\,dx$ $\dfrac{2\pi}{3}[(3x+1)^{\frac{1}{2}}]_0^5$ $\dfrac{2\pi}{3}[4-1]$ $[2\pi]$ Line: $(\pi)\int_0^5 (\dfrac{1}{100}x^2 - \dfrac{1}{5}x + 1)dx$ $(\pi)[\dfrac{1}{300}x^3 - \dfrac{1}{10}x^2 + x]_0^5$ $(\pi)[\dfrac{125}{300} - \dfrac{25}{10} + 5]$ $[\dfrac{35\pi}{12}]$ Volume $= \dfrac{35\pi}{12} - 2\pi = \dfrac{11\pi}{12}$	M1 A1A1 DM1 M1 A2,1 DM1 A1	[9]	Attempt $\int_0^5 y^2 dx$ (π not vital) (π not vital). 2nd A mark is for ÷ 3. Application of limits to *their* integral (in either integral). Limits 0 to 5 only. Attempt $\int_0^5 y^2 dx$ (π not vital) Also directly $-\dfrac{10}{3}(-\dfrac{1}{10}x+1)^3$ or $-\dfrac{10}{3}\left[(-\dfrac{1}{2}+1)^3 - 1^3\right]$ (π not vital) – applying limits to *their* integral Subtraction of *their* volumes co

Q6.

7	**(i)**	$\dfrac{3(1+2x)^{-1}}{-1} + (c)$ $y = \dfrac{3(1+2x)^{-1}}{-2} + (c)$ Sub (1, (1/2)) $\dfrac{1}{2} = \dfrac{3}{-6} + c \Rightarrow c = 1$	B1 B1(indep) M1 A1 [4]	 Division by 2 $y =$ necessary Dependent on c present Use of $y = mx + c$ etc. gets 0/4
	(ii)	$(1+2x)^2(>)9$ or $4x^2 + 4x - 8(>)0$ **OE** 1, −2 $x > 1, x < -2$ ISW	M1 A1 A1 [3]	

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Q7.

1	$\int\left(x^3+\frac{1}{x^3}\right)dx = \frac{x^4}{4}+\frac{x^{-2}}{-2}+c$	3 × B1 [3]	Allow unsimplified, 1 mark for each term, including "c"

Q8.

(b) $\int(3x-2)^5\,dx = \frac{(3x-2)^6}{6} \div 3\ (+c)$	B1 B1	B1 without "÷ 3". B1 for "÷ 3". (ignore (+c))
$\int_0^1(3x-2)^5\,dx = \left[\frac{(3x-2)^6}{18}\right]$	M1	Uses limits after integration.
Limits used correctly → −3½	A1 [4]	co

Q9.

4	**(i)** 3	B1	[1]	
	(ii) $f(x)=x^2-6x(+c)$	M1A1		Dependent on c present
	Subst $(3,-4)$	M1		cao
	$c=5 \rightarrow f(x)=x^2-6x+5$	A1	[4]	

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Q10.

10	**(i)** B $=(0,1)$ C $=(4,3)$	B1, B1	[2]	If B0B0 then SCB1 for both $y=1$ & $x=4$
	(ii) $\frac{\delta y}{\delta x}=\frac{1}{2}\times 2(1+2x)^{-\frac{1}{2}}$ Grad. of normal $=-3$ $y-3=-3(x-4)$ or $y=-3x+15$ oe	M1A1 B1 B1√	[4]	$-\frac{1}{2}$ required & at least one of $\frac{1}{2}\times 2$ for M1 Ft only from *their* C
	(iii) $y^2=1+2x\Rightarrow x=\frac{1}{2(y^2-1)}$ SOI $(\pi)\times\frac{1}{4}\times\int(y^4-2y^2+1)\delta y$ $(\pi)\times\frac{1}{4}\left[\frac{y^5}{5}-\frac{2y^3}{3}+y\right]$ $(\pi)\times\frac{1}{4}\left[\frac{1}{5}-\frac{2}{3}+1\right]$ $\frac{2}{15}\pi$	B1 M1 A1 DM1 A1	[5]	$\int x^2\delta y$, square $\frac{1}{2}(y^2-1)$ & attempt int″ Apply limits $0\rightarrow$ *their* 1 (from *their* B) cao SCB1 for $\int y^2\delta x\rightarrow\frac{\pi}{4}$ (scores 1/5)

Q11.

7 $\frac{dy}{dx}=5-\frac{8}{x^2}$, Normal $3y+x=17$		
(i) Gradient of line $=-⅓$ $\frac{dy}{dx}=3\rightarrow x=2,\ y=5$	B1 M1 DM1 A1 [4]	co Use of $m_1m_2=-1$ DM1 solution. A1 co.
(ii) $y=5x+8x^{-1}(+c)$ Uses (2, 5) $\rightarrow$ $c=-9$	B1 B1 M1 A1 [4]	co.co. doesn't need $+c$. Use of $+c$ following integration. co.