Functions 2

Q1.

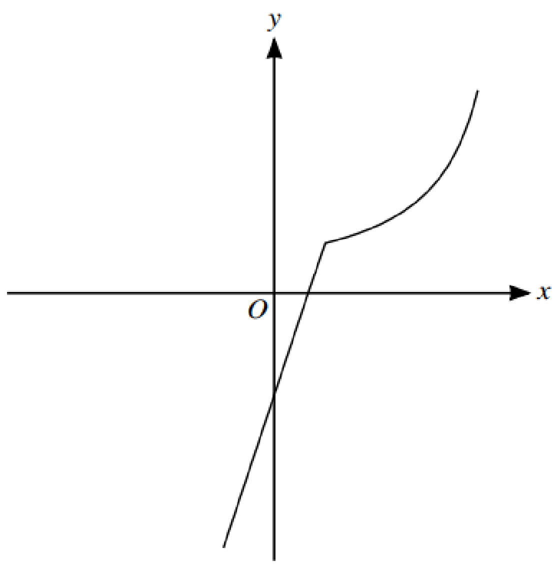
The function f is defined by $f : x \mapsto x^2 + 4x$ for $x \geqslant c$, where c is a constant. It is given that f is a one-one function.

(i) State the range of f in terms of c and find the smallest possible value of c. [3]

The function g is defined by $g : x \mapsto ax + b$ for $x \geqslant 0$, where a and b are positive constants. It is given that, when $c = 0$, $gf(1) = 11$ and $fg(1) = 21$.

(ii) Write down two equations in a and b and solve them to find the values of a and b. [6]

Q2.



The diagram shows the function f defined for $-1 \leqslant x \leqslant 4$, where

$$f(x) = \begin{cases} 3x - 2 & \text{for } -1 \leqslant x \leqslant 1, \\ \dfrac{4}{5 - x} & \text{for } 1 < x \leqslant 4. \end{cases}$$

(i) State the range of f. [1]

(ii) Copy the diagram and on your copy sketch the graph of $y = f^{-1}(x)$. [2]

(iii) Obtain expressions to define the function f^{-1}, giving also the set of values for which each expression is valid. [6]

Functions 2

Q3.

Functions f and g are defined by

$$f : x \mapsto 2x - 3, \quad x \in \mathbb{R},$$
$$g : x \mapsto x^2 + 4x, \quad x \in \mathbb{R}.$$

(i) Solve the equation $ff(x) = 11$. [2]

(ii) Find the range of g. [2]

(iii) Find the set of values of x for which $g(x) > 12$. [3]

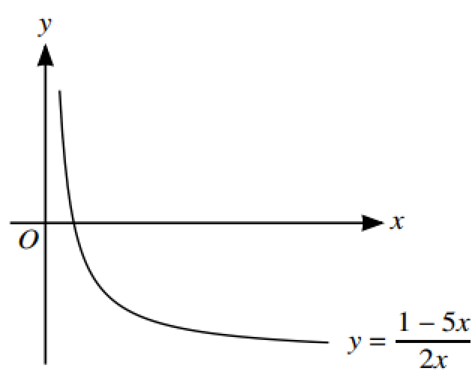
(iv) Find the value of the constant p for which the equation $gf(x) = p$ has two equal roots. [3]

Function h is defined by $h : x \mapsto x^2 + 4x$ for $x \geqslant k$, and it is given that h has an inverse.

(v) State the smallest possible value of k. [1]

(vi) Find an expression for $h^{-1}(x)$. [4]

Q4.



The diagram shows the graph of $y = f^{-1}(x)$, where f^{-1} is defined by $f^{-1}(x) = \dfrac{1-5x}{2x}$ for $0 < x \leqslant 2$.

(i) Find an expression for $f(x)$ and state the domain of f. [5]

(ii) The function g is defined by $g(x) = \dfrac{1}{x}$ for $x \geqslant 1$. Find an expression for $f^{-1}g(x)$, giving your answer in the form $ax + b$, where a and b are constants to be found. [2]

Functions 2

Q5.

Functions f and g are defined by

$$f : x \mapsto 10 - 3x, \quad x \in \mathbb{R},$$
$$g : x \mapsto \frac{10}{3 - 2x}, \quad x \in \mathbb{R},\ x \neq \tfrac{3}{2}.$$

Solve the equation $ff(x) = gf(2)$. [3]

Q6.

The function f is defined by $f : x \mapsto \dfrac{2}{3 - 2x}$ for $x \in \mathbb{R},\ x \neq \tfrac{3}{2}$.

(i) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g : x \mapsto 4x + a$ for $x \in \mathbb{R}$, where a is a constant.

(ii) Find the value of a for which $gf(-1) = 3$. [3]

(iii) Find the possible values of a given that the equation $f^{-1}(x) = g^{-1}(x)$ has two equal roots. [4]

Q7.

Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$
$$g(x) = x^2 + 4x + 13.$$

(a) By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x + p) + q$, where p and q are constants. [4]

(b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

Functions 2

Q8.

(a) Express $2x^2 - 8x + 14$ in the form $2[(x - a)^2 + b]$. [2]

The functions f and g are defined by

$$f(x) = x^2 \quad \text{for } x \in \mathbb{R},$$
$$g(x) = 2x^2 - 8x + 14 \quad \text{for } x \in \mathbb{R}.$$

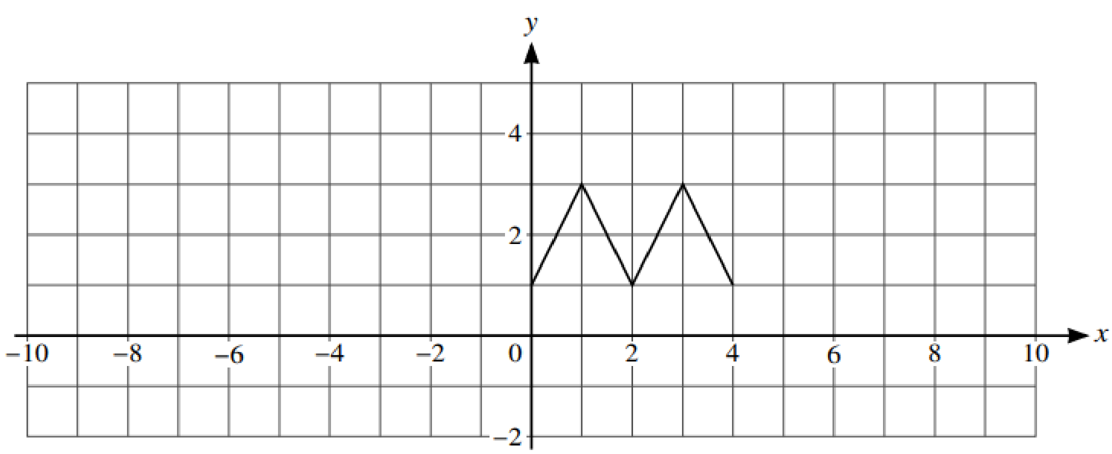
(b) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$, making clear the order in which the transformations are applied. [4]

Q9.

The graph with equation $y = f(x)$ is transformed to the graph with equation $y = g(x)$ by a stretch in the x-direction with factor 0.5, followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(a) The diagram below shows the graph of $y = f(x)$.

On the diagram sketch the graph of $y = g(x)$. [3]



(b) Find an expression for $g(x)$ in terms of $f(x)$. [2]
