Circular Measure

Pure Mathematics 1 (9709)

Degrees and Radians

$\boxed{180\ degrees\ = \pi\ radians}$

Degrees → Radians : $\times\ by\ \frac{\pi}{180^0}$

e.g. convert 720^0 to radians. $\cancel{720}^{4}{}^\circ \times \frac{\pi}{\cancel{180^\circ}_{1}} = 4\pi$

Radians → Degrees : $\times\ by\ \frac{180^0}{\pi}$

e.g. convert $\frac{5\pi}{6}$ radians to degrees $\frac{5\cancel{\pi}}{\cancel{6}_{1}} \times \frac{\cancel{180^\circ}^{30^\circ}}{\cancel{\pi}} = 150^\circ$

Common angles

degrees	radians
180^0	π
90^0	$\frac{1}{2}\pi$
45^0	$\frac{1}{4}\pi$
360^0	2π

degrees	radians
30^0	$\frac{1}{6}\pi$
60^0	$\frac{1}{3}\pi$
120^0	$\frac{2}{3}\pi$
150^0	$\frac{5}{6}\pi$

Common trigonometric ratios

θ	sin	tan	cos
30° $\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{\sqrt{3}}{2}$
45° $\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$
60° $\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{2}$

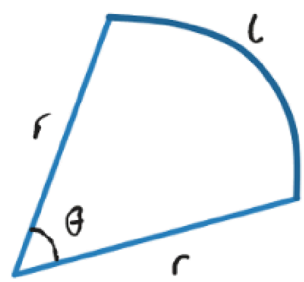
Arc Length

To find the arc length l of a sector of a circle, we can use the formula

$$\boxed{l = r\,\theta}$$

r - radius of the circle

θ - sector angle in radians



Areas

To find the area of a sector of a circle, we can use the formula

$$\boxed{A = \frac{1}{2}r^2\theta}$$

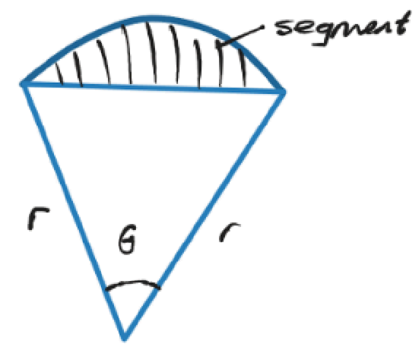
θ is in radians

To find the area of a segment of a circle, we can use the formula

$$A_{segment} = A_{sector} - A_{triangle}$$

$$A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

$$\boxed{A = \frac{1}{2}r^2(\theta - \sin\theta)}$$



Visit waltermaths.com for more learning resources.

COURSES WORKSHEETS TUTORING PATREON

WALTER MATHS

Circular Measure

Pure Mathematics 1 (9709)

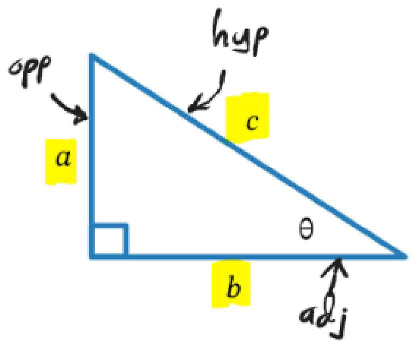
Plane Trigonometry

Trigonometric ratios:

$\cos\theta = \frac{b}{c}$ = adj/hyp

$\sin\theta = \frac{a}{c}$ = opp/hyp

$\tan\theta = \frac{a}{b}$ = opp/adj



$\boxed{a = c\sin\theta}$

opp = hyp sin θ

$\boxed{b = c\cos\theta}$

adj = hyp cos θ

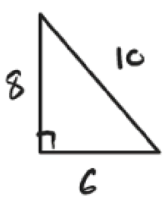
$\boxed{\text{SOH CAH TOA}}$

Pythagoras theorem:

$$\boxed{c^2 = a^2 + b^2}$$

Common Pythagorean triples:

hyp	opp ⇆ adj	
5	4	3
10	8	6
13	12	5
17	15	8
25	24	7



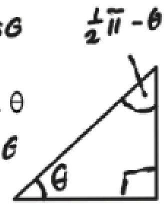
Complementary angle relations:

$\sin(90^0 - \theta) = \cos\theta$

$\sin(\frac{1}{2}\pi - \theta) = \cos\theta$

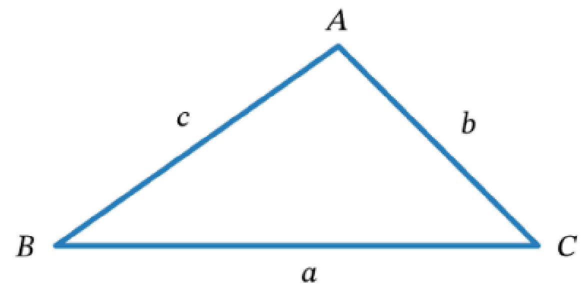
$\cos(90^0 - \theta) = \sin\theta$

$\cos(\frac{1}{2}\pi - \theta) = \sin\theta$



$\tan(90^0 - \theta) = \frac{1}{\tan\theta}$

$\tan(\frac{1}{2}\pi - \theta) = \frac{1}{\tan\theta}$



Sine Rule:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or

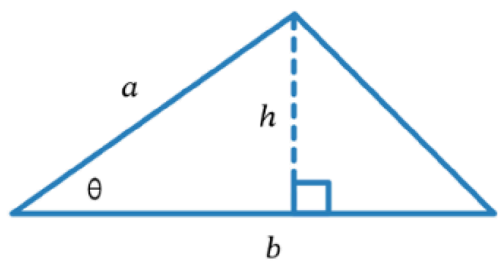
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc\cos\theta$$

$$b^2 = a^2 + c^2 - 2ac\cos\theta$$

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$



Area:

$\boxed{A = \frac{1}{2}bh}$ height known

or $\boxed{A = \frac{1}{2}ab\sin\theta}$ height unknown

Visit waltermaths.com for more learning resources.

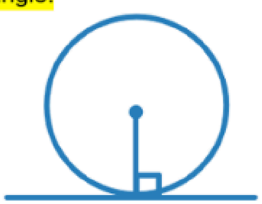
COURSES WORKSHEETS TUTORING PATREON

WALTER MATHS

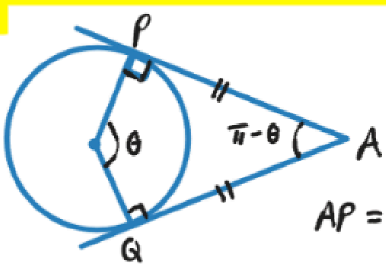
Circle Theorems and Properties

1. The angle in a semi-circle is a right angle.

2. The angle between a tangent and the radius at the point where the tangent touches the circle is a right angle.

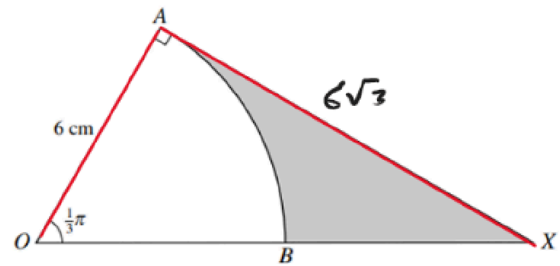


3. Two tangents drawn from a point to a circle are equal.



$AP = AQ$

Example 1



In the diagram, AB is an arc of a circle, centre O and radius 6 cm, and angle $AOB = \frac{1}{3}\pi$ radians. The line AX is a tangent to the circle at A, and OBX is a straight line.

(i) Show that the exact length of AX is $6\sqrt{3}$ cm. [1]

Find, in terms of π and $\sqrt{3}$,

(ii) the area of the shaded region, [3]

(iii) the perimeter of the shaded region. [4]

(i) $\tan\frac{1}{3}\pi = \frac{AX}{6}$

$AX = 6\tan\frac{1}{3}\pi$

$= 6\sqrt{3}$

(ii) $A_{shaded} = A_{AOAX} - A_{sector\ OAB}$

$= \frac{1}{2}\times 6\times 6\sqrt{3} - \frac{1}{2}\times 6^2\times\frac{1}{3}\pi$

$= 18\sqrt{3} - 6\pi$

(iii) $P = arc AB + AX + BX$

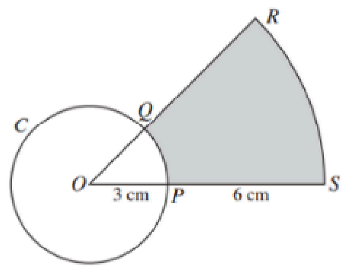
$= 6\times\frac{1}{3}\pi + 6\sqrt{3} + \left(\sqrt{6^2+(6\sqrt{3})^2} - 6\right)$

$= 2\pi + 6\sqrt{3} + (\sqrt{144} - 6) = 2\pi + 6\sqrt{3} + 6$ – Ans

Circular Measure

Pure Mathematics 1 (9709)

Example 2



The diagram shows a circle C with centre O and radius 3 cm. The radii OP and OQ are extended to S and R respectively so that ORS is a sector of a circle with centre O. Given that $PS = 6$ cm and that the area of the shaded region is equal to the area of circle C,

(i) show that angle $POQ = \frac{1}{4}\pi$ radians, [3]

(ii) find the perimeter of the shaded region. [2]

(i) $A_{shaded} = A_{circle\ C}$

$\frac{1}{2} \times 9^2 \theta - \frac{1}{2} \times 3^2 \theta = \pi \times 3^2$

$\frac{1}{2}\theta(81 - 9) = 9\pi$

$36\theta = 9\pi$

$\theta = \frac{9}{36}\pi = \frac{1}{4}\pi$

(ii) $P = 6 + 6 + 3\theta + 9\theta$

$= 12 + 12\theta$

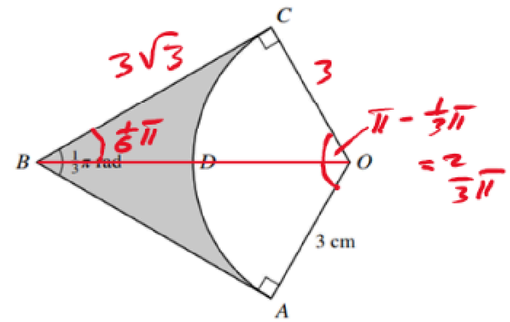
$= 12 + 12 \times \frac{1}{4}\pi$

$= 12 + 3\pi$

Example 3

$\tan \frac{1}{6}\pi = \frac{3}{BC}$

$BC = \frac{3}{\tan \frac{1}{6}\pi}$



In the diagram, $OADC$ is a sector of a circle with centre O and radius 3 cm. AB and CB are tangents to the circle and angle $ABC = \frac{1}{3}\pi$ radians. Find, giving your answer in terms of $\sqrt{3}$ and π,

(i) the perimeter of the shaded region, [3]

(ii) the area of the shaded region. [3]

(i) $P = \underbrace{BC + AB}_{} + \text{arc } ADC$

$= 2 \times \frac{3}{\tan \frac{1}{6}\pi} + 3 \times \frac{2}{3}\pi$

$= 2 \times 3\sqrt{3} + 2\pi$

$= 6\sqrt{3} + 2\pi$

(ii) $A_{shaded} = A_{OABC} - A_{sector\ OADC}$

$\Rightarrow A = 2 \times \frac{1}{2} \times 3 \times 3\sqrt{3} - \frac{1}{2} \times 3^2 \times \frac{2}{3}\pi$

$= 9\sqrt{3} - 3\pi$