Functions 1 MS

Q1.

9	(i) $2x^2 - 12x + 7 = 2(x-3)^2 - 11$	$3 \times$ B1 [3]	B1 for each value – accept if a, b, c not specifically quoted.
	(ii) Range of f $\geqslant -11$	B1√ [1]	√ to his "c". allow $>$ or $\geqslant$.
	(iii) $2x^2 - 12x + 7 < 21$		
	$\rightarrow 2x^2 - 12x - 14$ or	M1	3-term quadratic to 0 or $2(x-3)^2 < 32$
	$2(x-3)^2 < 32$		
	$\rightarrow$ end-values of 7 or -1	A1	Correct end-values
	$\rightarrow -1 < x < 7$	A1 [3]	co
	(iv) gf(x) $= 2(2x^2 - 12x + 7) + k = 0$	M1 A1	Puts f into g. co.
	Use of $b^2 - 4ac$	M1	Used correctly with quadratic
	$\rightarrow 24^2 - 16(14 + k)$		
	$\rightarrow k = 22$	A1 [4]	co.

Q2.

3	f : $x \mapsto 4x - 2x^2$,		
	g : $x \mapsto 5x + 3$.		
	(i) Turning point at $x = 1$.	M1	Calculus or completing the square etc.
	Range is $\leqslant 2$.	A1 [2]	Condone $<$ instead of $\leqslant$.
	(ii) gf(x) $= 5(4x - 2x^2) + 3$	B1	For putting f into g.
	$= k$ and use of $b^2 - 4ac$	M1	Setting to k, using $b^2 - 4ac$
	$\rightarrow k = 13$	A1 [3]	co

Functions 1 MS

Q3.

10	$f: x \mapsto 2x^2 - 8x + 14$			
	(i) $y + kx = 12$, Sim Eqns.	M1		Complete elimination of y (or x)
	$\rightarrow 2x^2 - 8x + kx + 2 = 0$	A1		
	Use of $b^2 - 4ac$	M1		Uses $b^2 - 4ac$ on eqn = 0, no "x" in a, b, c.
	$\rightarrow (k-8)^2 = 16 \rightarrow k = 12$ or 4.	A1	[4]	co.co
	(ii) $2x^2 - 8x + 14 = 2(x-2)^2 + 6$	B1×3	[3]	
	(iii) Range of f $\geqslant 6$.	B1√	[1]	√ for c or from calculus.
	(iv) Smallest $A = 2$	B1√	[1]	√ to answer to **(ii)**.
	(v) Makes x the subject	M1		Could interchange x, y first.
	Order of operations correct.	M1		Order must be correct.
	$g^{-1}(x) = \sqrt{\dfrac{x-6}{2}} + 2$	A1	[3]	co

Q4.

3	$f: x \mapsto 2x + 3$,			
	$g: x \mapsto x^2 - 2x$,			
	$gf(x) = (2x+3)^2 - 2(2x+3)$	M1		Must be f into g, not g into f.
	$= 4x^2 + 8x + 3$	A1		co
	$= 4(x+1)^2 - 1$	3 × B1√	[5]	Allow all these as √ for either fg or gf.

Q5.

7	**(i)** $(x-2)^2$	M1		Must be "-2" $\pm k$
	$(x-2)^2 + 3$	A1		co
	$f(x) > 3$	B1√	[3]	ft on *their* '3'
	(ii) $x - 2 = (\pm)\sqrt{y-3}$	M1		$\pm$ not required for M mark
	$f^{-1}(x) = 2 + \sqrt{x-3}$	A1		$f(x)$ + removal of minus sign needed
	domain is $x > 3$	B1√	[3]	ft domain of f^{-1} = range of f or for f^{-1}
	(iii) $h(x) = x^2 + 3$	B1	[1]	co

Functions 1 MS

Q6.

10 $f : x \mapsto 3x - 4$ $\qquad g : x \mapsto 2(x-1)^3 + 8$			
(i) $fg(2) = f(10) = 26$ $\qquad\qquad\qquad f^{-1}(x)$	M1 A1 [2]	Must use g first, then f. co
(ii)	B1 B1 B1 [3]	$y = f(x)$ correct in 1st, 4th quadrants. $y = f^{-1}(x)$ correct in 1st, 2nd quadrants. $y = x$ marked, or quoted.
(iii) $g'(x) = 6(x-1)^2$ $g'(x) > \rightarrow$ no turning points $\rightarrow$ g is 1 : 1, g has an inverse.	B1 B1√ B1√ [3]	co allow only for incorrect "6" following from incorrect "6"
(iv) $f^{-1}(x) = \dfrac{x+4}{3}$ Attempt at making x Order correct. $-8, \div 2, \sqrt[3]{\ }, +1$ $g^{-1}(x) = \sqrt[3]{\dfrac{x-8}{2}} + 1$	B1 M1 M1 A1 [4]	co May change x and y first. Must all be correct, but allow for $+8, -1$ co as function of x, not y.
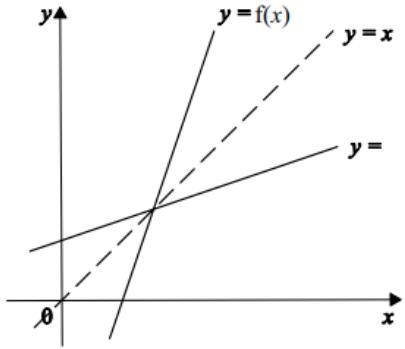
Q7.

2	[Stretch] [factor 2, x direction (or y-axis invariant)]	*B1 DB1	
	[Translation or Shift] [1 unit in y direction] **or** [Translation/Shift] $\begin{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{bmatrix}$	B1B1	Accept transformations in either order. Allow (0, 1) for the vector
		4	

Functions 1 MS

Q8.

3(a)	$(y) = f(-x)$	B1
		1
3(b)	$(y) = 2f(x)$	B1
		1
3(c)	$(y) = f(x+4) - 3$	B1 B1
		2

Q9.

2(a)	Translation $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	B1	Allow shift and allow by 1 in *x*-direction or [parallel to/on/in/ along/against] the *x*-axis or horizontally. 'Translation by 1 to the right' only, scores B0
	Stretch	B1	Stretch. **SC B2** for amplitude doubled.
	Factor 2 in *y*-direction	B1	With/by **factor** 2 in *y*-direction or [parallel to/on/in/along/against] the *y*-axis or vertically or with *x* axis invariant 'With/by factor 2 upwards' only, scores B0. Accept SF as an abbreviation for scale factor.
		3	**Note:** Transformations can be in either order
2(b)	$[-\sin 6x][+ 15x]$ or $[\sin(-6x)][+ 15x]$ OE	B1 B1	Accept an unsimplified version. ISW. B1 for each correct component – square brackets indicate each required component.
			If B0, **SC B1** for either $\sin(-2x) + 5x$ or $-\sin(2x) + 5x$ or $\sin 6x - 15x$ or $\sin\left(-\dfrac{2}{3}x\right) + \dfrac{5}{3}x$
		2	