Trigonometry 1

Q1.

(i) Show that the equation

$$3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$$

can be written in the form $\tan x = -\frac{3}{4}$. [2]

(ii) Solve the equation $3(2\sin x - \cos x) = 2(\sin x - 3\cos x)$, for $0^\circ \leqslant x \leqslant 360^\circ$. [2]

Q2.

(i) Show that the equation $2\sin x \tan x + 3 = 0$ can be expressed as $2\cos^2 x - 3\cos x - 2 = 0$. [2]

(ii) Solve the equation $2\sin x \tan x + 3 = 0$ for $0^\circ \leqslant x \leqslant 360^\circ$. [3]

Q3.

(i) Prove the identity $\dfrac{\sin x \tan x}{1 - \cos x} \equiv 1 + \dfrac{1}{\cos x}$. [3]

(ii) Hence solve the equation $\dfrac{\sin x \tan x}{1 - \cos x} + 2 = 0$, for $0^\circ \leqslant x \leqslant 360^\circ$. [3]

Q4

Prove the identity

$$\tan^2 x - \sin^2 x \equiv \tan^2 x \sin^2 x.$$ [4]

Q5.

(i) Show that the equation $2\tan^2\theta \sin^2\theta = 1$ can be written in the form

$$2\sin^4\theta + \sin^2\theta - 1 = 0.$$ [2]

(ii) Hence solve the equation $2\tan^2\theta \sin^2\theta = 1$ for $0^\circ \leqslant \theta \leqslant 360^\circ$. [4]

Trigonometry 1

Q6.

(i) Prove the identity $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 \equiv \frac{1-\cos\theta}{1+\cos\theta}$. [3]

(ii) Hence solve the equation $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 = \frac{2}{5}$, for $0° \leqslant \theta \leqslant 360°$. [4]

Q7.

(i) Given that

$$3\sin^2 x - 8\cos x - 7 = 0,$$

show that, for real values of x,

$$\cos x = -\tfrac{2}{3}.$$ [3]

(ii) Hence solve the equation

$$3\sin^2(\theta + 70°) - 8\cos(\theta + 70°) - 7 = 0$$

for $0° \leqslant \theta \leqslant 180°$. [4]

Q8.

(i) Prove the identity $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$. [2]

(ii) Solve the equation $\frac{2}{\sin x \cos x} = 1 + 3\tan x$, for $0° \leqslant x \leqslant 180°$. [4]

Q9.

(i) Prove the identity $\tan^2\theta - \sin^2\theta \equiv \tan^2\theta \sin^2\theta$. [3]

(ii) Use this result to explain why $\tan\theta > \sin\theta$ for $0° < \theta < 90°$. [1]

Q10.

(i) Solve the equation $\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 360°$. [5]

(ii) How many solutions has the equation $\sin 2x + 3\cos 2x = 0$ for $0° \leqslant x \leqslant 1080°$? [1]
