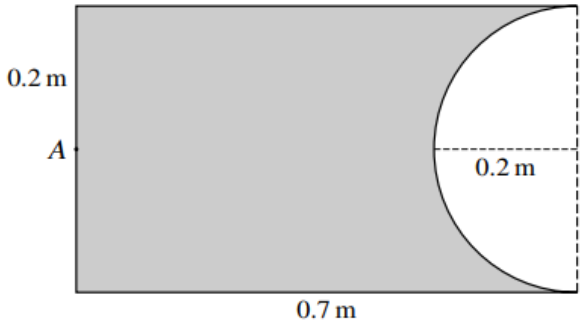
Equilibrium of a Rigid Body 1

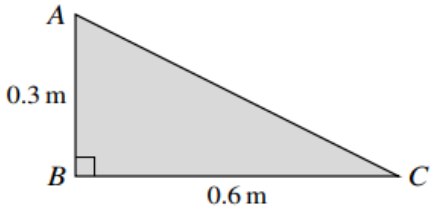
Q1.



The diagram shows the cross-section through the centre of mass of a uniform solid object. The object is a cylinder of radius 0.2 m and length 0.7 m, from which a hemisphere of radius 0.2 m has been removed at one end. The point A is the centre of the plane face at the other end of the object. Find the distance of the centre of mass of the object from A. [5]

[The volume of a hemisphere is $\frac{2}{3}\pi r^3$.]

Q2.



ABC is a uniform lamina in the form of a triangle with $AB = 0.3$ m, $BC = 0.6$ m and a right angle at B (see diagram).

(i) State the distances of the centre of mass of the lamina from AB and from BC. [2]

The lamina is freely suspended at B and hangs in equilibrium.

(ii) Find the angle between AB and the horizontal. [2]

A force of magnitude 12 N is applied along the edge AC of the lamina in the direction from A towards C. The lamina, still suspended at B, is now in equilibrium with AB vertical.

(iii) Calculate the weight of the lamina. [3]

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Q3.

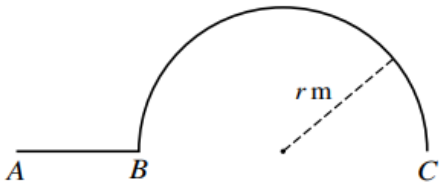

Fig. 1

Fig. 1 shows an object made from a uniform wire of length 0.8 m. The object consists of a straight part AB, and a semicircular part BC such that A, B and C lie in the same straight line. The radius of the semicircle is r m and the centre of mass of the object is 0.1 m from line ABC.

(i) Show that $r = 0.2$. [3]

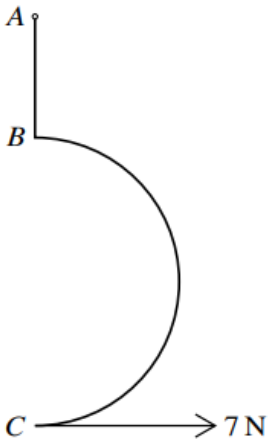

Fig. 2

The object is freely suspended at A and a horizontal force of magnitude 7 N is applied to the object at C so that the object is in equilibrium with ABC vertical (see Fig. 2).

(ii) Calculate the weight of the object. [3]

The 7 N force is removed and the object hangs in equilibrium with ABC at an angle of $\theta°$ with the vertical.

(iii) Find θ. [6]

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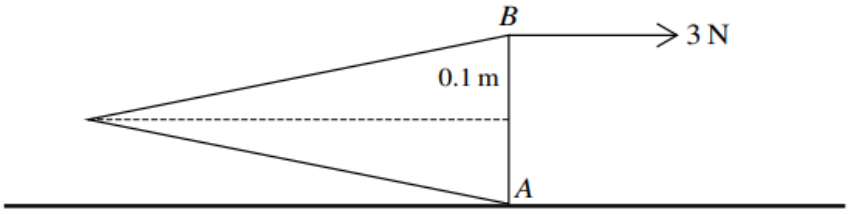
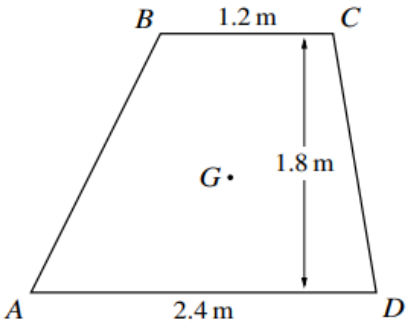
Q4.



A uniform solid cone has weight 5 N and base radius 0.1 m. AB is a diameter of the base of the cone. The cone is held in equilibrium, with A in contact with a rough horizontal surface and AB vertical, by a force applied at B. This force has magnitude 3 N and acts parallel to the axis of the cone (see diagram). Calculate the height of the cone. [3]

Q5.



$ABCD$ is a uniform lamina in the shape of a trapezium which has centre of mass G. The sides AD and BC are parallel and 1.8 m apart, with $AD = 2.4$ m and $BC = 1.2$ m (see diagram).

(i) Show that the distance of G from AD is 0.8 m. [4]

The lamina is freely suspended at A and hangs in equilibrium with AD making an angle of 30° with the vertical.

(ii) Calculate the distance AG. [2]

With the lamina still freely suspended at A a horizontal force of magnitude 7 N acting in the plane of the lamina is applied at D. The lamina is in equilibrium with AG making an angle of 10° with the downward vertical.

(iii) Find the two possible values for the weight of the lamina. [5]

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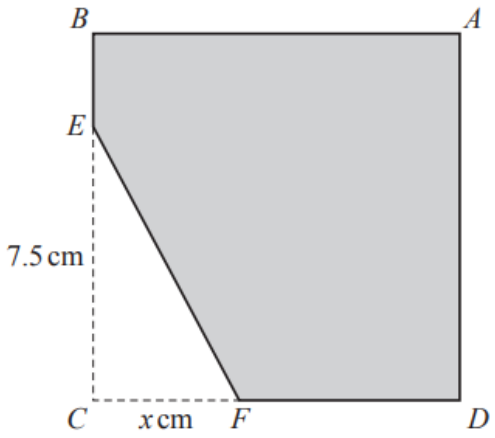
Q6.



A uniform square lamina $ABCD$ has sides of length 10 cm. The point E is on BC with $EC = 7.5$ cm, and the point F is on DC with $CF = x$ cm. The triangle EFC is removed from $ABCD$ (see diagram). The centre of mass of the resulting shape $ABEFD$ is a distance $\bar{x}$ cm from CB and a distance $\bar{y}$ cm from CD.

(a) Show that $\bar{x} = \dfrac{400 - x^2}{80 - 3x}$ and find a corresponding expression for $\bar{y}$. [4]

The shape $ABEFD$ is in equilibrium in a vertical plane with the edge DF resting on a smooth horizontal surface.

(b) Find the greatest possible value of x, giving your answer in the form $a + b\sqrt{2}$, where a and b are constants to be determined. [3]
