Polar Coordinates 1

Q1. The curves C_1 and C_2 have polar equations

$$r = \theta + 2 \quad \text{and} \quad r = \theta^2$$

respectively, where $0 \leqslant \theta \leqslant \pi$.

(i) Find the polar coordinates of the point of intersection of C_1 and C_2. [2]

(ii) Sketch C_1 and C_2 on the same diagram. [2]

(iii) Find the area bounded by C_1, C_2 and the line $\theta = 0$. [3]

Q2. The curve C has polar equation

$$r = \left(\tfrac{1}{2}\pi - \theta\right)^2,$$

where $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. Draw a sketch of C. [3]

Find the area of the region bounded by C and the initial line, leaving your answer in terms of π. [3]

Q3. Draw a sketch of the curve C whose polar equation is $r = \theta$, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. [2]

On the same diagram draw the line $\theta = \alpha$, where $0 < \alpha < \frac{1}{2}\pi$. [1]

The region bounded by C and the line $\theta = \frac{1}{2}\pi$ is denoted by R. Find the exact value of α for which the line $\theta = \alpha$ divides R into two regions of equal area. [4]

Q4. The curve C has polar equation

$$r = a\left(1 - e^{-\theta}\right),$$

where a is a positive constant and $0 \leqslant \theta < 2\pi$.

(i) Draw a sketch of C. [3]

(ii) Show that the area of the region bounded by C and the lines $\theta = \ln 2$ and $\theta = \ln 4$ is

$$\tfrac{1}{2}a^2\left(\ln 2 - \tfrac{13}{32}\right).$$ [4]

Q5. The curve C has polar equation $r = 2\cos 2\theta$. Sketch the curve for $0 \leqslant \theta < 2\pi$. [4]

Find the exact area of one loop of the curve. [4]

Polar Coordinates 1

Q6. The curves C_1 and C_2 have polar equations

$$C_1: \quad r = a,$$
$$C_2: \quad r = 2a\cos 2\theta, \text{ for } 0 \leqslant \theta \leqslant \tfrac{1}{4}\pi,$$

where a is a positive constant. Sketch C_1 and C_2 on the same diagram. [3]

The curves C_1 and C_2 intersect at the point with polar coordinates (a, β). State the value of β. [1]

Show that the area of the region bounded by the initial line, the arc of C_1 from $\theta = 0$ to $\theta = \beta$, and the arc of C_2 from $\theta = \beta$ to $\theta = \frac{1}{4}\pi$ is

$$a^2\left(\tfrac{1}{6}\pi - \tfrac{1}{8}\sqrt{3}\right).$$ [4]

Q7. The curve C has polar equation $r = 3 + 2\cos\theta$, for $-\pi < \theta \leqslant \pi$. The straight line l has polar equation $r\cos\theta = 2$. Sketch both C and l on a single diagram. [3]

Find the polar coordinates of the points of intersection of C and l. [4]

The region R is enclosed by C and l, and contains the pole. Find the area of R. [6]

Q8. The curve C has polar equation $r = 1 + \sin\theta$ for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$. Draw a sketch of C. [2]

The area of the region enclosed by the initial line, the half-line $\theta = \frac{1}{2}\pi$, and the part of C for which θ is positive, is denoted by A_1. The area of the region enclosed by the initial line, and the part of C for which θ is negative, is denoted by A_2. Find the ratio $A_1 : A_2$, giving your answer correct to 1 decimal place. [8]

Q9. The curve C has cartesian equation

$$(x^2 + y^2)^2 = a^2(x^2 - y^2),$$

where a is a positive constant. Show that C has polar equation

$$r^2 = a^2\cos 2\theta.$$ [2]

Sketch C for $-\pi < \theta \leqslant \pi$. [2]

Find the area of the sector between $\theta = -\frac{1}{4}\pi$ and $\theta = \frac{1}{4}\pi$. [3]

Find the polar coordinates of all points of C where the tangent is parallel to the initial line. [7]
