Momentum 2 MS

Q1.

3	Use conservation of momentum, e.g.:	$mv_A + 2mv_B = mu$	B1		
	Use restitution (must be consistent with prev. eqn.):	$v_A - v_B = -eu$	B1		
	Find speed of *B* after striking barrier (ignore sign):	$v_B' = \frac{1}{2} v_B$	M1		
	Relate K.E. before and after collision:	$(\frac{1}{2}mu^2)/9 = \frac{1}{2}mv_A{}^2 + \frac{1}{2}(2m)v_B{}'^2$	M1		
	EITHER: Solve first two eqns for v_A and v_B (A.E.F):	$v_A = \frac{1}{3}(1 - 2e)u,\ v_B = \frac{1}{3}(1 + e)u$	M1 A1		
	Substitute for v_A, v_B' in KE eqn:	$u^2/9 = (1 - 2e)^2u^2/9 + \frac{1}{2}(1 + e)^2 u^2/9$	A1		
	Simplify and solve for *e*:	$9e^2 - 6e + 1 = 0,\ \ e = \frac{1}{3}$	M1 A1		
	OR: Use $v_A + 2v_B = u$ in KE eqn to give e.g.:	$81v_A{}^2 - 18uv_A + u^2 = 0$ *or* $81v_B{}^2 - 72uv_B + 16u^2 = 0$	(M1 A1)		
	Solve for v_A and v_B:	$v_A = u/9$ *and* $v_B = 4u/9$	(A1)		
	Find *e* from restitution eqn:	$e = (4u/9 - u/9)/u = \frac{1}{3}$	(M1 A1)	9	**9**

Q2.

2	Use conservation of momentum, e.g.:	$4mv_A + \lambda mv_B = 4mu$	B1		
	Use restitution (must be consistent with prev. eqn.):	$v_A - v_B = -\frac{1}{2}u$	B1		
	Solve for v_B:	$4(v_B - \frac{1}{2}u) + \lambda v_B = 4u$			
	(or verify eqns are satisfied by this v_B)	$v_B = 6u / (\lambda + 4)$ **A.G.**	M1 A1	4	
	Use conservation of momentum, e.g.:	$\lambda mw_B + mw_C = \lambda mv_B$	B1		
	Use restitution (must be consistent with prev. eqn.):	$w_B - w_C = -\frac{1}{2}v_B$	B1		
	Eliminate w_B:	$(1 + \lambda)w_C = (1 + \frac{1}{2})\lambda v_B$	M1		
	Put $w_C = u$, substitute for v_B and solve for λ:	$(1 + \lambda) = 9\lambda / (\lambda + 4)$			
		$\lambda^2 - 4\lambda + 4 = 0,\ \lambda = 2$	M1 A1	5	**9**

Momentum 2 MS

Q3.

1 (i)	Use conservation of momentum, e.g.	$mv_A + kmv_B = mu + \frac{2}{3}kmu$ or $v_A + kv_B = u(1 + \frac{2}{3}k)$ B1		
	Use restitution (4/5 on wrong side is M0; signs inconsistent with prev. eqn is A0):	$v_A - v_B = -(4/5)(u - \frac{2}{3}u)$ or $v_A - v_B = -4u/15$ M1 A1		
	Solve for v_A (allow verification):	$(1+k)v_A = u(1 + \frac{2}{3}k - 4k/15)$ $v_A = u(2k+5)/5(k+1)$ **A.G.** M1 A1 $[v_B = u(10k+19)/15(k+1)]$	5	
(ii)	Equate impulse to momentum change for A:	$mu - (2/5)mu = mv_A$ $3/5 = (2k+5)/5(k+1)$, $k = 2$ M1 A1		
	OR B:	$\frac{2}{3}kmu + (2/5)mu = kmv_B$ $\frac{2}{3}k + (2/5) = k(10k+19)/15(k+1)$ $10k^2 + 16k + 6 = 10k^2 + 19k$, $k = 2$ (M1 A1)	2	7

Q4.

5	For A & B use conservation of momentum, e.g.: (*m* may be omitted here and below)	$3mv_A + 2mv_B = 3mu$	M1		
	Use Newton's law of restitution (consistent signs):	$v_B - v_A = eu$	M1		
	Combine to find v_B :	$v_B = 3(1+e)u/5$	A1		
	For B & C use conservation of momentum, e.g.:	$2mv_B' + mv_C = 2mv_B$	M1		
	Use Newton's law of restitution (consistent signs):	$v_C - v_B' = e'v_B$	M1		
	Combine to find v_C and v_B':	$v_C = 2(1+e')v_B/3$ $= 2(1+e)(1+e')u/5$ AG	A1		
		$v_B' = (2-e')v_B/3$ $= (1+e)(2-e')u/5$	A1	7	
	Find ratios or values of v_A, v_B', v_C from momentum:	$3v_A = 2v_B' = v_C$ $[= u]$	B1		
	Find e from first collision eqns, e.g.: (or find e' and then use $3v_A = 2v_B'$)	$v_A = (3-2e)u/5 = u/3$ or $v_B = \frac{1}{2}(3u - u)$ or $(\frac{1}{3} + e)u$ $= 3(1+e)u/5$, $e = \frac{2}{3}$	M1 A1		
	Find e' from second collision eqns, e.g.:	$2v_B' = v_C$ so $2(2-e') = 2(1+e')$ or $v_C = 2(1+\frac{2}{3})(1+e')u/5 = u$ or $v_B' = (1+\frac{2}{3})(2-e')u/5 = u/2$ $e' = \frac{1}{2}$	M1 A1	5	12

Momentum 2 MS

Q5.

2 (i)	*EITHER:* Find comps. of speed after colln. at E:	$v \cos 45°$ // to wall *and*		
		$\tfrac{3}{4} v \sin 45° \perp$ to wall	**M1 A1**	
	Relate v to u, or v^2 to u^2:	$\sqrt{\{(v / \sqrt{2})^2 + (\tfrac{3}{4} v / \sqrt{2})^2\}} = \tfrac{1}{4} u$	**M1**	
		$(5/4\sqrt{2})\, v = \tfrac{1}{4} u$	**A1**	
		$v = (\sqrt{2}/5)\, u$ A.G	**A1**	
	OR: Relate angle β after colln. to u, v:	$\tfrac{1}{4} u \cos \beta = v \cos 45°$ *and*		
		$\tfrac{1}{4} u \sin \beta = \tfrac{3}{4} v \sin 45°$	**(M1 A1)**	
	Find $\tan \beta$, or β:	$\tan \beta = \tfrac{3}{4}$ *or* $\beta = 36{\cdot}9°$	**(A1)**	
	Eliminate β from either eqn. above, e.g.:	$\tfrac{1}{4} u \times (4/5) = v / \sqrt{2}$	**(M1)**	
		$v = (\sqrt{2}/5)\, u$ A.G.	**(A1)**	[5]
(ii)	Relate comps. of speed // to wall after colln. at D:	$v \cos 75° = u \cos \alpha$	**M1**	
	Find $\cos \alpha$:	$\cos \alpha = (\sqrt{2}/5) \cos 75°$ [$= 0{\cdot}0732$]	**A1**	
	Find α:	$\alpha = 85{\cdot}8°$ *or* $1{\cdot}50$ rads	**A1**	
	Relate comps. of speed $\perp$ to wall after colln. at D:	$v \sin 75° = eu \sin \alpha$	**M1**	
	Find e:	$e = (\sqrt{2}/5) \sin 75° / \sin \alpha$		
		or $= \tan 75° / \tan \alpha = 0{\cdot}274$	**A1**	[5]

Q6.

3(i)	$mv_A + mv_B = mu$ (AEF)	***M1**	Use conservation of momentum (allow $v_A + v_B = u$)
	$v_B - v_A = \tfrac{2}{3} u$	***M1**	Use Newton's restitution law (consistent LHS signs)
	$v_B = 5u/6$	**A1**	Combine to find v_B
	$w_B = \tfrac{1}{3} v_B = 5u/18$ AG	**B1**	Verify speed w_B of B after collision with wall (ignore sign)
	Total:	**4**	

3(ii)	$v_A = u / 6$	**DA1**	Find v_A (dependent on above ***M1 *M1**)
	EITHER: $(d - x) / v_A = d / v_B + x / w_B$ (AEF)	**(M1 A1**	*EITHER:* Equate times in terms of reqd. distance x
	$6(d - x) = 1{\cdot}2\, d + 3{\cdot}6\, x$	**M1 A1)**	Substitute for speeds to formulate an eqn. in x
	OR: $x_A = (d/v_B)\, v_A = (6d/5u)\, u/6 = 0{\cdot}2\, d$	**(M1**	*OR:* Find dist. x_A moved by A when B reaches wall
	$t_2 = (0{\cdot}8\, d) / (v_A + w_B) = 9d/5u$	**M1 A1**	Find remaining time t_2
	$y_A = v_A t_2 = 0{\cdot}3\, d$ *or* $y_B = w_B t_2 = 0{\cdot}5\, d$	**A1)**	Find remaining distance moved by A or B
	OR2: $x_A = (d/v_B)\, v_A = (6d/5u)\, u/6 = 0{\cdot}2\, d$	**(M1**	*OR2:* Find dist. x_A moved by A when B reaches wall
	$(0{\cdot}8\, d - x) / v_A = x / w_B$ *or* $0{\cdot}8\, d/(v_A + w_B) = x/w_B$	**M1 A1**	Equate remaining times to formulate an eqn. in x
	$4{\cdot}8\, d - 6\, x = 3{\cdot}6\, x$ *or* $1{\cdot}8\, d = 3{\cdot}6\, x$	**A1)**	
	$x = \tfrac{1}{2} d$	**A1**	Find x
	Total:	**6**	

Momentum 2 MS

Q7.

6(a)	Let v be speed of rebound from 1st collision: Energy loss: $\frac{1}{2}mv^2 = \frac{2}{5} \times \frac{1}{2}mu^2$, $v^2 = \frac{2}{5}u^2$	**B1**	Energy loss.
	$v\cos\alpha = u\cos\theta$ $v\sin\alpha = eu\sin\theta$	**B1**	Both.
	Combine to form equation in e only $\frac{2}{5} = \frac{1}{5} + e^2 \times \frac{4}{5}$	**M1**	$v^2 = (u\cos\theta)^2 + (eu\sin\theta)^2$
	$e = \frac{1}{2}$	**A1**	
		4	
6(b)	$\tan\alpha = e\tan\theta$, so $\tan\alpha = 1$, $\alpha = 45°$	**B1**	
	For 2nd collision $w\cos\beta = v\cos(180 - 60 - \alpha)$ $w\sin\beta = ev\sin(180 - 60 - \alpha)$	**M1**	Both. May be implied by the A1.
	$\tan\beta = e\tan(120 - their\,\alpha)$	**M1**	Divide to find β.
	$\beta = 61.8°$	**A1**	
		4	

Q8.

6(a)	Let speed of A after collision be $\rightarrow v_A$ and speed of B perpendicular to line of centres be $\downarrow v$ Along line of centres: $mu - km\frac{5}{8}u\cos\alpha = mv_A$	**M1**	Momentum.
	NEL: $0 - v_A = e\left(\frac{5}{8}u\cos\alpha + u\right)$	**M1**	NEL
	So $u - \frac{5}{8}ku\cos\alpha = -\frac{2}{3}\left(\frac{5}{8}u\cos\alpha + u\right)$	**M1**	Solve.
	Substitute for cos, to give $k = 4$	**A1**	
		4	
6(b)	$v_B = \frac{5}{8}u\sin\alpha = \frac{3}{8}u$	**B1**	Velocity perpendicular to line of centres
	$v_A = -u$	**B1 FT**	
	KE before $= \frac{1}{2}mu^2 + \frac{1}{2}km\left(\frac{5}{8}u\right)^2 = \frac{1}{2}mu^2 + \frac{25}{32}mu^2 = \frac{41}{32}mu^2$ KE after $= \frac{1}{2}mv_A^2 + \frac{1}{2}kmv_B^2 = \frac{1}{2}mu^2 + 2m\frac{9}{64}u^2 = \frac{25}{32}mu^2$	**M1**	NOTE: KE before and after for A is unchanged. Both.
	Loss $= mu^2\left(\frac{41}{32} - \frac{25}{32}\right) = \frac{1}{2}mu^2$	**A1**	
		4	