Trigonometry 2 MS

Q1.

5	(i)	$\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$. Divides top and bottom by cos θ $\rightarrow \quad \frac{t-1}{t+1}$	B1 [1]	Answer given.
	(ii)	$\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}=\frac{1}{6}\tan\theta$ $\rightarrow \frac{t-1}{t+1}=\frac{t}{6}$ $\rightarrow t^2-5t+6=0$ $\rightarrow t=2$ or $t=3$ $\rightarrow \theta=63.4^\circ$ or 71.6°	B1 M1 A1 A1 [4]	Using the identity. Forms a 3 term quadratic with terms all on same side. co co

Q2.

4	(i)	$\tan\theta=1/3$ $\theta=18.4^\circ$ only	M1 A1 [2]	Ignore solns. outside range 0→180
	(ii)	**tan**$2x=(\pm)1/\sqrt{3}$ Must be sq. root soi	M1	$\sin 2x=(\pm)1/2$ **or** $\cos 2x=(\pm)\sqrt{3/2}$ using $c^2+s^2=1$. Not tan $x=(\pm)\frac{1}{\sqrt{3}}$ etc.
		$(x)=15$ $(x)=$ any correct second value (75, 105, 165) $(x)=$ cao	A1 A1√ A1 [4]	ft for (90 ± their 15) or (180 – their 15) All four correct. Extra solns in range 1

Q3.

4	(i)	$4\cos^2\theta+15\sin\theta=0$	M1	Replace tan θ by $\frac{\sin\theta}{\cos\theta}$ and multiply by sin θ or equivalent
		$4(1-s^2)+15s=0\rightarrow 4\sin^2\theta-15\sin\theta-4=0$	M1A1 [3]	Use $c^2=1-s^2$ and rearrange to **AG** (www)
	(ii)	$\sin\theta=-1/4$ $\theta=194.5$ or 345.5	B1 B1B1√ [3]	Ignore other solution Ft from 1st solution, SC B1 both angles in rads (3.39 and 6.03)

Trigonometry 2 MS

Q4.

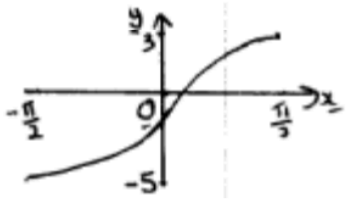
4	(i)	$\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)^2=\left(\frac{1}{s}-\frac{c}{s}\right)^2$ $\frac{(1-c)^2}{s^2}=\frac{(1-c)^2}{1-c^2}$ $=\frac{(1-c)(1-c)}{(1-c)(1+c)}$ or $\frac{(1-c)^2}{(1-c)(1+c)}$ $\equiv\frac{1-\cos x}{1+\cos x}$	**M1** **M1** **A1** **A1** [4]	Use of tan = sin/cos Use of $s^2 = 1 - c^2$ ag
	(ii)	$\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)^2=\frac{2}{5}$ $\frac{1-\cos x}{1+\cos x}=\frac{2}{5}\rightarrow\cos x\frac{3}{7}$ $\rightarrow$ $x = 1.13$ or 5.16	 **M1** **A1 A1**√ [3]	 Making cos$x$ the subject $2\pi - 1^{st}$ answer.

Q5.

4	(a)	$3x=-\sqrt{3}/2$ $x=\frac{-\sqrt{3}}{6}$ oe	**M1** **A1** **[2]**	Accept −0.866 at this stage Or $\frac{-3}{6\sqrt{3}}$ or $\frac{-1}{2\sqrt{3}}$
	(b)	$(2\cos\theta-1)(\sin\theta-1)=0$ $\cos\theta=1/2$ or $\sin\theta=1$ $\theta=\pi/3$ *or* $\pi/2$	**M1** **A1** **A1A1** **[4]**	Reasonable attempt to factorise and solve Award B1B1 www Allow 1.05, 1.57. SCA1for both 60°, 90°

Trigonometry 2 MS

Q6.

11	**(i)**	$f: x \to 4\sin x - 1$ for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ Range $-5 \leqslant f(x) \leqslant 3$	**B1** **B1** [2]	–5 and 3 Correct range
	(ii)	$4s - 1 = 0 \to s = \frac{1}{4} \to x = 0.253$ $x = 0 \to y = -1$	**M1 A1** **B1** [3]	Makes sin$x$ subject. Degrees **M1 A0**, (14.5^o)
	(iii)		**B1**√ **B1** [2]	Shape from their range in (i) Flattens, curve.
	(iv)	range $-\frac{1}{2}\pi \leqslant f^{-1}(x) \leqslant \frac{1}{2}\pi$ domain $-5 \leqslant x \leqslant 3$ Inverse $f^{-1}(x) = \sin^{-1}\left(\frac{x+1}{4}\right)$	**B1** **B1**√ **M1 A1** [4]	√ on part (i) (only for 2 numerical values) Correct order of operations

Q7.

7	**(i)**	$\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta} \equiv \frac{4}{\sin\theta\tan\theta}$ $\text{LHS} = \frac{1+2c+c^2-(1-2c+c^2)}{(1-c)(1+c)}$ $= \frac{4c}{1-c^2}$ $= \frac{4c}{s^2}$ $= \frac{4}{ts}$ **AG**	**M1** **A1 A1** **A1** [4]	Attempt at combining fractions. A1 for numerator. A1 denominator Essential step for award of A1
	(ii)	$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3.$ $\to s \times \frac{4}{ts} = 3 \ (\to t = \frac{4}{3})$ $\theta = 53.1°$ and $233.1°$	**M1** **A1 A1**√ [3]	Uses part (i) to eliminate "s" correctly. √ for 180° + 1st answer.

Trigonometry 2 MS

Q8.

8	**(i)**	$3\sin^2 x - \cos^2 x + \cos x = 0$ Use $s^2 = 1 - c^2$ and simplify to 3-term quad $\cos x = -3/4$ and 1 $x = 2.42$ (allow 0.77π) or 0 (extra in range max 1)	**M1** **M1** **A1** **A1A1** [5]	Multiply by $\cos x$ Expect $4c^2 - c - 3 = 0$ SC1 for 0.723 (or 0.23π), π following $4c^2 + c - 3 = 0$
	(ii)	$2x = 2\pi - \textit{their}2.42$ or 360 – 138.6 x = 1.21 (0.385π), 1.93 (0.614/5π), 0, π (3.14) (extra max 1)	**B1**√ **B1B1** [3]	Expect $2x = 3.86$ Any 2 correct B1. Remaining 2 correct B1. SCB1for all 69.3, 110.7, 0, 180 (degrees) SCB1 for .361, π/2, 2.78 after $4c^2 + c - 3 = 0$

Q9.

6	**(i)**	$\cos^4 x = (1 - \sin^2 x)^2 \quad = 1 - 2\sin^2 x + \sin^4 x$ AG	**B1**	[1]	Could be LHS to RHS or vice versa
	(ii)	$8\sin^4 x + 1 - 2\sin^2 x + \sin^4 x = 2(1 - \sin^2 x)$ $9\sin^4 x = 1$ $x = 35.3^o$ (or any correct solution) Any correct second solution from 144.7°, 215.3°, 324.7° The remaining 2 solutions	**M1** **A1** **A1** **A1**√ **A1**	[5]	Substitute for $\cos^4 x$ and $\cos^2 x$ or OR sub for $\sin^4 x \rightarrow 3\cos^2 x = 2$ $\rightarrow \cos x = (\pm)\sqrt{2/3}$ Allow the first 2 **A1** marks for radians (0.616, 2.53, 3.76, 5.67)

Q10.

2	**(i)**	$2\sin 2x = 6\cos 2x$ $\tan 2x = k$ $\rightarrow \tan 2x = 3$ or $k = 3$	**M1** **A1**	[2]	Expand and collect as far as $\tan 2x$ = a constant from sin ÷ cos soi cwo
	(ii)	$x =$ $(\tan^{-1}(\textit{their}\, k)) \div 2$ (71.6° or −108.4°) ÷2 $x = 35.8°, -54.2°$ $x = 0.624^c, -0.946^c$ $x = 0.198\pi^c, -0.301\pi^c$	**M1** **A1 A1**√	[3]	Inverse then ÷2. soi. √ on 1st answer +/ − 90° if in given range but no extra solutions in the given range. Both SR A1A0