Differentiation 2 MS

Q1.

8 **(i)** $A = 2xr + \pi r^2$	**B1**	
$2x + 2\pi r = 400$ $(\Rightarrow x = 200 - \pi r)$	**B1**	
$A = 400r - \pi r^2$	**M1A1** **[4]**	Subst & simplify to **AG** (www)
(ii) $\frac{dA}{dr} = 400 - 2\pi r$	**B1**	Differentiate
$= 0$	**M1**	Set to zero and attempt to find r
$r = \frac{200}{\pi}$ oe	**A1**	
$x = 0 \Rightarrow$ no straight sections **AG**	**A1**	
$\frac{d^2A}{dr^2} = -2\pi$ (<0) Max	**B1** **[5]**	Dep on -2π, or use of other valid reason

Q2.

6		
(i) Sim triangles $\frac{y}{16-x} = \frac{12}{16}$ (or trig)	M1	Trig, similarity or eqn of line
$\rightarrow y = 12 - \frac{3}{4}x$	A1	(could also come from eqn of line)
$A = xy = 12x - \frac{3}{4}x^2$.	A1 [3]	ag – check working.
(ii) $\frac{dA}{dx} = 12 - \frac{6x}{4}$	B1	
$= 0$ when $x = 8$. $\rightarrow A = 48$.	M1 A1	Sets to 0 + solution.
This is a Maximum. From −ve quadratic or 2nd differential.	B1 [4]	Can be deduced without any working. Allow even if '48' incorrect.

Q3.

9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$	M1A1	Attempt differentiation & set to zero
$x + 2 = \pm k$	DM1	Attempt to solve
$x = -2 \pm k$	A1	cao
$\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$	M1	Attempt to differentiate again
	M1	Sub their x value with k in it into $\frac{d^2y}{dx^2}$
When $x = -2 = k$, $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (>0) min	A1	Only 1 of bracketed items needed for each
When $x = -2 - k$, $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (<0) max	A1 [8]	but $\frac{d^2y}{dx^2}$ and x need to be correct.

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Q4.

4 $\frac{dy}{dx} = \left[-2\times4(3x+1)^{-3}\right]\times[3]$	**B1B1**	$[-2 \times 4u^{-3}] \times [3]$ is B0B1 unless resolved
When $x=-1$, $\frac{dy}{dx}=3$	**B1**	
When $x=-1$, $y=1$ soi	**B1**	
$y-1=3(x+1)$ $(\rightarrow y=3x+4)$	**B1** √ **[5]**	Ft on *their* '3' only (not $-\frac{1}{3}$). Dep on diffn

Q5.

12 (i) $y=\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}+(c)$ oe	**B1B1**	Attempt to integrate
$\frac{2}{3}=\frac{16}{3}-4+c$	**M1**	Sub $\left(4,\frac{2}{3}\right)$. Dependent on c present
$c=-\frac{2}{3}$	**A1** **[4]**	
(ii) $\frac{1}{2}x^{-\frac{1}{2}}+\frac{1}{2}x^{-\frac{3}{2}}$ oe	**B1B1** **[2]**	
(iii) $x^{\frac{1}{2}}-x^{-\frac{1}{2}}=0\rightarrow\frac{x-1}{\sqrt{x}}=0$	**M1**	Equate to zero and attempt to solve
$x=1$	**A1**	
When $x=1$, $y=\frac{2}{3}-2-\frac{2}{3}=-2$	**M1A1**	Sub. *their* '1' into *their* 'y'
When $x=1$, $\frac{d^2y}{dx^2}(=1)>0$ Hence minimum	**B1** **[5]**	Everything correct on final line. Also dep on correct (ii). Accept other valid methods

Q6.

4 $y=\frac{12}{3-2x}$		
(i) Differential $=-12(3-2x)^{-2}\times-2$	B1 B1 [2]	co co (even if 1st B mark lost)
(ii) $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}=0.4\div0.15$	M1	Chain rule used correctly (AEF)
$\rightarrow\frac{24}{(3-2x)^2}=\frac{8}{3}$	M1	Equates their $\frac{dy}{dx}$ with their $\frac{8}{3}$ or $\frac{3}{8}$
$\rightarrow x=0$ or 3	A1 A1 [4]	co co

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Q7.

4	$u = 2x(y - x)$ and $x + 3y = 12$, $u = 2x\left(\dfrac{12 - x}{3} - x\right)$	M1 A1	Expresses u in terms of x
	$= 8x - \frac{8x^2}{3}$		
	$\dfrac{\mathrm{d}u}{\mathrm{d}x} = 8 - \dfrac{16x}{3}$	M1	Differentiate candidate's quadratic, sets to 0 + attempt to find x, or other valid method
	$= 0$ when $x = 1\frac{1}{2}$	A1	
	$\rightarrow (y = 3\frac{1}{2})$ $\rightarrow u = 6$	A1 [5]	Complete method that leads to u Co

Q8.

8 (i)	$-(x+1)^{-2} - 2(x+1)^{-3}$	**M1A1** **A1** **[3]**	M1 for recognisable attempt at differentn. Allow $\dfrac{-x^2 - 4x - 3}{(x+1)^4}$ from Q rule. (A2,1,0)
(ii)	$f'(x) < 0$ hence decreasing	**B1** **[1]**	Dep. on *their* (i) < 0 for $x > \ 1$
(iii)	$\dfrac{-1}{(x+1)^2} - \dfrac{2}{(x+1)^3} = 0$ **or** $\dfrac{-x^2 - 4x - 3}{(x+1)^4} = 0$	**M1***	Set $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ to 0
	$\dfrac{-(x+1) - 2}{(x+1)^3} = 0 \rightarrow -x - 1 - 2 = 0$ or $-x^2 - 4x - 3 = 0$	**M1** **Dep***	OR mult by $(x+1)^3$ or $(x+1)^5$ (i.e.×mult) $\times$ multn $\rightarrow -(x+1)^3 - 2(x+1)^2 = 0$
	$x = -3,\ y = -1/4$	**A1A1** **[4]**	$(-3, -1/4)$ www scores 4/4

Q9.

7(i)	$f'(x) = \left[\frac{3}{2}(4x+1)^{1/2}\right]$ [4]	**B1B1**	Expect $6(4x+1)^{1/2}$ but can be unsimplified.
	$f''(x) = 6 \times 1/2 \times (4x+1)^{-1/2} \times 4$	**B1√**	Expect $12(4x+1)^{-1/2}$ but can be unsimplified. Ft from *their* $f'(x)$.
	Total:	**3**	
7(ii)	$f(2)$, $f'(2)$, $kf''(2) = 27, 18, 4k$ OR 12	**B1B1√B1√**	Ft dependent on attempt at differentiation
	$27/18 = 18/4k$ oe OR $kf''(2) = 12 \Rightarrow k = 3$	**M1A1**	
	Total:	**5**	

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Q10.

10(a)	$f''(x) = -\left(\frac{1}{2}x + k\right)^{-3}$	**B1**	
	$f''(2) > 0 \Rightarrow -(1+k)^{-3} > 0$	**M1**	Allow for solving *their* $f''(2) > 0$
	$k < -1$	**A1**	WWW
		3	
10(b)	$\left[f(x) = \int\left(\left(\frac{1}{2}x-3\right)^{-2} - (-2)^{-2}\right)dx =\right] \left\{\frac{\left(\frac{1}{2}x-3\right)^{-1}}{-1\times\frac{1}{2}}\right\}\left\{-\frac{x}{4}\right\}$	**B1 B1**	Allow $-2\left(\frac{1}{2}x+k\right)^{-1}$ OE for 1st B1 and $-(1+k)^{-2}x$ OE for 2nd B1
	$3\frac{1}{2} = 1 - \frac{1}{2} + c$	**M1**	Substitute $x = 2$, $y = 3\frac{1}{2}$ into *their* integral with c present.
	$f(x) = \frac{-2}{\left(\frac{1}{2}x-3\right)} - \frac{x}{4} + 3$	**A1**	OE
		4	
10(c)	$\left(\frac{1}{2}x-3\right)^{-2} - (-2)^{-2} = 0$	**M1**	Substitute $k = -3$ and set to zero.
	leading to $\left(\frac{1}{2}x-3\right)^{2} = 4$ $\left[\frac{1}{2}x - 3 = (\pm)2\right]$ leading to $x = 10$	**A1**	
	$(10, -\frac{1}{2})$	**A1**	Or when $x = 10$, $y = -1 - 2\frac{1}{2} + 3 = -\frac{1}{2}$
	$f''(10)\left[= -(5-3)^{-3} \rightarrow\right] < 0 \rightarrow$ MAXIMUM	**A1**	WWW
		4	