Continuous Random Variables 1 MS

Q1

6		Integrate to find F(x) for $1 \le x \le 3$: State F(x) for other intervals of x:	$F(x) = \frac{1}{2}(x - 1)$ $0\ (x < 1),\ \ 1\ (x > 3)$	B1 B1	2	
	(i)	Relate dist. fn. G(y) of Y to X: (working may be omitted) Differentiate to find g(y):	$G(y) = P(Y < y) = P(X^3 < y)$ $= P(X < y^{1/3}) = F(y^{1/3})$ $= \frac{1}{2}(y^{1/3} - 1)$ $g(y) = y^{-2/3}/6\ \ (1 \le y \le 27)$ [= 0 otherwise]	 M1 A1 B1	3	
	(ii)	Find expected value of Y (or X^3): Find variance of Y:	$E(Y) = \int_1^{27} y\,(y^{-2/3}/6)\,dy$ $= [y^{4/3}/8]_1^{27}$ *or* $[x^4/8]_1^3$ $= (81 - 1)/8 = 10$ $E(Y^2) = \int_1^{27} y^2\,(y^{-2/3}/6)\,dy$ $= [y^{7/3}/14]_1^{27}$ *or* $[x^7/14]_1^3$ $= (2187 - 1)/14 = 1093/7$ $\text{Var}(Y) = E(Y^2) - 10^2$ $= 393/7$ *or* $56{\cdot}1[4]$	 B1 M1 A1	3	**[8]**

Q2.

9	Integrate f(x) to find F(x) for $-a \le x \le a$ (A.E.F.): (Finding $F(x) = x/2a$ earns M1 A0) State F(x) for other intervals: Relate dist. fn. G(y) of Y to X for central interval: (working may be omitted; ignore other intervals) Find k from $P(Y \ge k) = \frac{1}{4}$ with $a = 4$: (Using $G(k) = \frac{1}{4}$ can earn M1)	$F(x) = x/2a + c = (x + a)/2a$ $F(x) = 0\ (x < -a),\ F(x) = 1\ (x > a)$ $G(y) = P(Y < y) = P(e^X < y)$ $= P(X < \ln y) = F(\ln y)$ $= (\ln y + a)/2a;\ \ (e^{-a} \le y \le e^a)$ $1 - G(k) = \frac{1}{4},\ (\ln k)/2a + \frac{1}{2} = \frac{3}{4}$ $\ln k = 2,\ \ k = e^2$ *or* $7{\cdot}39$	M1 A1 B1 M1 A1; A1 M1 A1 A1	 3 3 3	 **[9]**

Q3.

7	Find $P(X \ge \frac{3}{4})$: State deduction about upper quartile Q_3 (dep *A1): Express cum. dist. fn. G of Y in terms of X: Relate to F: Simplify: State G in full (√ on previous result):	$1 - F(\frac{3}{4}) = 1 - \frac{1}{2}[(\frac{3}{4})^3 + 1]$ $= 37/128$ *or* $0{\cdot}289$ $Q_3 > \frac{3}{4}$ $G(y) = P(Y \leqslant y) = P(-\sqrt{y} \leqslant X \leqslant \sqrt{y})$ $= F(\sqrt{y}) - F(-\sqrt{y})$ $= \frac{1}{2}[(\sqrt{y})^3 + 1] - \frac{1}{2}[(-\sqrt{y})^3 + 1]$ $= y^{3/2}$ $0\ (y < 0),\ y^{3/2}\ (0 \leqslant y \leqslant 1),\ 1\ (y > 1)$	M1 *A1 B1 M1 A1 M1 A1 A1√	 (3) (5)	 **[8]**

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Q4.

9	Find k for which $P(X \geqslant k) = 0{\cdot}6$: $\quad 0{\cdot}6 = 1 - F(k)$	M1		
	$= 1 - (k/8 - ¼)$	M1		
	$k = 26/5$ *or* $5{\cdot}2$	A1	3	
	Find G(y) from $Y = 2 \ln X$ for $2 \leqslant x \leqslant 10$: $\quad G(y) = P(Y < y) = P(2 \ln X < y)$			
	(allow < or ≤ throughout) $\quad = P(X < e^{y/2}) = F(e^{y/2})$			
	(result may be stated) $\quad = e^{y/2}/8 - ¼ \; (2 \ln 2 \leqslant y \leqslant 2 \ln 10)$			
	or $(\ln 4 \leqslant y \leqslant \ln 100)$			
	or $(1{\cdot}39 \leqslant y \leqslant 4{\cdot}61)$	M1 A1		
	State G(y) for other values of x: $\quad 0 \; (y < 2 \ln 2)$ *and* $1 \; (y > 2 \ln 10)$	B1	3	
	Find g(y) for $2 \ln 2 \leqslant y \leqslant 2 \ln 10$: $\quad g(y) = e^{y/2}/16$	M1 A1		
	Sketch positive exponential for $2 \ln 2 \leqslant y \leqslant 2 \ln 10$	B1		
	Show $g(y) = 0$ on either side of this interval	B1	4	**10**

Q5.

10	Find F(x) for $1 \leqslant x \leqslant 3$: $\quad F(x) = \frac{1}{2}(x - 1)$	B1		
	Find G(y) from $Y = X^3$ for $1 \leqslant x \leqslant 3$: $\quad G(y) = P(Y < y) = P(X^3 < y)$ $= P(X < y^{\frac{1}{3}}) = F(y^{\frac{1}{3}})$			
	(result may be stated) $\quad = \frac{1}{2}(y^{\frac{1}{3}} - 1); \; (1 \leqslant y \leqslant 27)$	M1 A1; B1		
	State G(y) for other values of y: $\quad 0 \; (y < 1)$ *and* $1 \; (y > 27)$	B1	5	
	Find g(y) for $1 \leqslant y \leqslant 27$ (√ on G(y)): $\quad g(y) = \frac{y^{-\frac{2}{3}}}{6}$ *or* $\frac{1}{6y^{\frac{2}{3}}}$	B1√		
	Sketch g(y) for $1 \leqslant y \leqslant 27$ with $g(y) = 0$ on either side of this interval	B1 B1	3	
	Find mean of Y: $\quad E(Y) = \int y\, g(y)\, dy = \int (\frac{y^{\frac{1}{3}}}{6})\, dy$			
	(no need to find median = 8) $\quad = \left[\frac{y^{\frac{4}{3}}}{8}\right]_1^{27} = 10$	M1 A1		
	Find probability Y lies between median and mean:			
	$G(10) - G(8)$ *or* $\left\vert G(10) - \frac{1}{2} \right\vert$			
	$= \frac{1}{2}\left(10^{\frac{1}{3}} - 8^{\frac{1}{3}}\right)$			
	(2 s.f. sufficient) $\quad$ *or* $\left\vert \frac{1}{2}\left(10^{\frac{1}{3}} - 1\right) - \frac{1}{2} \right\vert = 0{\cdot}077$ [2]			
		M1 A1	4	**[12]**

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Q6.

8(i)	$F(x) = \int f(x)\,dx = x^2/8 - x/4$ [+ c]	M1	Find or state distribution function F(x) for $2 \leqslant x \leqslant 4$ using F(2) = 0 or F(4) = 1 to find c if necessary
	$= x^2/8 - x/4$ *or* $\{(x-1)^2 - 1\}/8$ (AEF)	A1	State F(x) for other values of x
	$F(x) = 0$ ($x < 2$), $F(x) = 1$ ($x > 4$)	A1	
	Total:	3	
8(ii)	*EITHER:* $G(y) = P(Y < y) = P((X-1)^3 < y)$ $= P(X < 1 + y^{1/3}) = F(1 + y^{1/3})$ $= (1 + y^{1/3})^2/8 - (1 + y^{1/3})/4$ *or* $(y^{2/3} - 1)/8$	(M1 A1)	Find or state G(y) for $2 \leqslant x \leqslant 4$ from $Y = (X-1)^3$ (allow < or $\leqslant$ throughout)
	OR: Use $x = 1 + y^{1/3}$ to find $f(x) = \frac{1}{4} y^{1/3}$ *and* $dx/dy = \frac{1}{3} y^{-2/3}$	(M1 A1)	Find f(x) *and* dx/dy for use in $g(y) = f(x) \times \lvert dx/dy \rvert$
	$g(y)$ [$= G'(y)$] $= (1/12)\, y^{-1/3}$ *or* $1 / (12\, y^{1/3})$	A1	Find g(y) in simplified form
	for $1 \leqslant y \leqslant 27$ [g(y) = 0 otherwise]	A1	State corresponding range of y for G(y) *or* g(y)
	Total:	4	
8(iii)	$(m^{2/3} - 1)/8 = \frac{1}{2}$	M1	Find median value m of Y from G(m) = ½
	$m^{2/3} = 5$, $m = \sqrt{125}$ *or* $5\sqrt{5}$ *or* 11·2	M1 A1	
	Total:	3	

Q7.

9(i)	*EITHER:* $F(x) = \int f(x)\,dx = (1/20)(3x - 2\sqrt{x}$ [+ c]) $c = -1$ so $F(x) = (1/20)(3x - 2\sqrt{x} - 1)$	M1	Find or state distribution function F(x) for $1 \leqslant x \leqslant 9$ (may be implied by G(y))
	or $(3/20)\,x - (1/10)\sqrt{x} - 1/20$ $G(y)$ [$= P(Y < y) = P(\sqrt{X} < y) = P(X < y^2)$]	A1	Find or state G(y) from $Y = \sqrt{X}$ for $1 \leqslant x \leqslant 9$ or $1 \leqslant y \leqslant 3$
	$= F(y^2)$ $= (1/20)(3y^2 - 2y - 1)$	M1	Allow A1√ as FT on expression found for F(x)
	or $(3/20)\,y^2 - (1/10)\,y - 1/20$	A2	Verify g(y) (differentiation may be implied)
	$g(y) = G'(y) = (1/10)(3y - 1)$ **AG** [for $1 \leqslant y \leqslant 3$, g($y$) = 0 otherwise]	M1 A1	**SC** Missing/incorrect c can earn M1 M1 A1√ M1 (max 4/7)
		7	
	OR: Use of $g(y) = f(x) \times \lvert dx / dy \rvert$	(*M1)	Reference to standard result required (not in syllabus)
	$f(x) = (1/20)(3 - 1/y)$ (dep *M1)	(M1 A1)	Find f(x) using $x = y^2$
	$dx/dy = 2y$ (dep *M1)	(M1 A1)	Find dx/dy using $x = y^2$
	$g(y) = f(x) \times dx/dy = (1/10)(3y - 1)$ **AG** [for $1 \leqslant y \leqslant 3$, g($y$) = 0 otherwise]	(M1 A1)	
		(7)	
9(ii)	$E(Y) = (1/10)\int(3y^2 - y)\,dy$	M1	Find mean of Y from $\int y\, g(y)\, dy$
	$= (1/10)\,[y^3 - \frac{1}{2} y^2]_1^3 = 11/5$ *or* 2·2	A1	
		2	